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LEHIGH UNIVERSITY

NASA RESEARCH GRANT NSG4005:
AIRCRAFT MODEL PROTOTYPES WHICH HAVE SPECIFIED
HANDLING-QUALITY TIME HISTORIES

(NASA-CR-146138) AIRCRAFT MODEL PROTOTYPES WHICH HAVE SPECIFIED HANDLING-QUALITY TIME HISTORIES Final Technical Report (Lehigh Univ.) 150 p HC $6.00 CSCL 01C

N76-16064

Unclas

G3/05 09973

TECHNICAL REPORT

BY

S. H. JOHNSON
PRINCIPAL INVESTIGATOR

JANUARY 1976

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

ABSTRACT

Several techniques for obtaining linear constant-coefficient airplane models from specified handling-quality time histories are discussed. One technique, the pseudodata method, solves the basic problem, yields specified eigenvalues, and accommodates state-variable transfer-function zero suppression. The algebraic equations to be solved are bilinear, at worst. The disadvantages are reduced generality and no assurance that the resulting model will be airplanelike in detail.

The method is fully illustrated for a fourth-order stability-axis small-motion model with three lateral handling-quality time histories specified. The Fortran program which obtains and verifies the model is included and fully documented.

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SYMBOLS

$\underline{a}$	vector containing all of the elements of $\underline{A}$
$\underline{A}$	plant matrix with elements a_{ij}
$\underline{b}$	input distribution vector with elements b_i
$\underline{C}$	coefficients matrix with elements c_{ij}
$\underline{c}$	final-value vector with elements c_i
C_n^*	normalized longitudinal handling-quality criterion
$C^*(t)$	longitudinal handling-quality criterion
$D(\ )$	denominator polynomial
D_n^*	normalized lateral handling-quality criterion
$D^*(t)$	lateral handling-quality criterion
d	differential operator $d \equiv \frac{d(\ )}{dt}$
$\underline{G}$	output measurement matrix, matrix transfer function
$\tilde{\underline{G}}$	augmented output measurement vector matrix
$\underline{h}$	input/output coupling vector
$\tilde{\underline{h}}$	augmented input/output coupling vector
ℓ	pilot station to vehicle C.G. distance
$\underline{N}(\ )$	numerator polynomials vector
p_n	normalized roll-rate handling-quality time history
$p(t)$	roll rate
q_{co}	cross-over dynamic pressure in D^* definition
$r(t)$	yaw rate
s	Laplace transformation variable
$\underline{T}$	similarity transformation matrix
$u(t)$	forcing function

V nominal airspeed

$\underset{\sim}{x}$ state vector with elements $x_i(t)$

$\underset{\sim}{y}$ output vector with elements $y_i(t)$

$\hat{y}$ specified response vector with elements $\hat{y}_i(t)$

$\tilde{y}$ augmented output vector

β_n normalized sideslip handling-quality time history

$\beta(t)$ sideslip

δ_a aileron deflection

δ_ε elevator deflection

$\underset{\sim}{\Lambda}$ eigenmatrix

$\hat{\Lambda}$ specified eigenmatrix

$\underset{\sim}{\lambda}$ eigenvalue vector with elements λ_j

$\phi(t)$ roll angle

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INTRODUCTION

Aircraft control systems have been designed in the past to meet stability, apparent damping, and sensitivity criteria. Subsequently, pilot comments and ratings, using the Cooper-Harper rating scale, graded the performance of the pilot-airplane-flight-control-system combination. The C-H scale emphasizes

TAKEOFF

CRUISE

GROSS MANEUVERING AND AEROBATICS

FORMATION FLIGHT

TRACKING

GROUND-CONTROLLED APPROACH

LANDING APPROACH [1]*

Experience has shown that the design criteria are often a subset of the handling-quality criteria. In order to design flight-control systems which achieve low (desirable) C-H ratings, some explicit consideration of handling qualities must be incorporated into the design process. The work begun by the Boeing Airplane Company and extended by the McDonnell-Douglas Company has resulted in quantitative specifications which are intended to insure "good" aircraft handling qualities. These specifications take the form of envelopes within which selected time-histories must fall, e.g., normalized sideslip, normalized roll rate, and two blended quantities, C^* and D^*, containing the acceleration cues at the pilot station.

An evolutionary process has provided the basic design of most aircraft control systems. It is often the case that the selection of numerical values

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*The numbers in square brackets refer to bibliography entries.

for the many parameters in a control system is more difficult than the establishment of the control system structure and modes of operation. To overcome this "tuning" problem, a model-matching optimization technique can be implemented on a large digital computer. Such a program adjusts specified parameters of a simulated airplane-flight-control-system combination so as to minimize some measure of the dynamic differences between the closed-loop airplane and a low-order model of a desirable prototype airplane. As a participant in the ASEE-NASA Summer Faculty Fellowship Program in the summers of 1974 and 1975, the principal investigator, working in the Vehicle Dynamics and Control Division of the NASA Flight Research Center, Edwards, California, developed a method of translating handling-quality time-history specifications into prototype aircraft models. These models are suitable for subsequent use with an FRC model-matching program: CONOPT.

The connection between specified time histories which fall within the established envelopes and numerical values for adjustable parameters onboard the aircraft is, in principle, established. The method is explained in the following sections and the lateral-motion case demonstrated.

PROBLEM STATEMENT

The NASA Flight Research Center computer program CONOPT, which resulted from the addition of the MIT Model Performance Index Design Program OPT to the in-house control system analysis program CONTROL, is a model-matching package. The user specifies the model in transfer-function form. The Model PI technique [2] requires that the model transfer function, if it has any zeros, should have the same number of excess poles over zeros as the closed-loop

airplane control system. If it has no zeros then the number of poles of the model transfer function should be equal to or less than the number of excess poles over zeros of the closed-loop airplane. Another condition on the model transfer function is that it must have reasonable eigenvalues. Thus the model is a linear, constant-coefficient ordinary differential equation in operator notation,

$$D(d)x(t) = N(d)u(t) \quad , \tag{1}$$

with loose bounds on the roots of,

$$D(\lambda_i) = 0 \quad , \tag{2}$$

and constraints on the coefficients of N(d). In order to obtain the entire matrix transfer function in one operation the model will be represented in state variable form,

$$d\underline{x} = \underline{\underline{A}}\underline{x} + \underline{b}u \quad . \tag{3}$$

The problem is to determine an $\underline{\underline{A}}$ and $\underline{b}$ combination which satisfies the loose bounds and constraints above and which describes a low-order prototype airplane model with "good" handling qualities. FRC program CONTROL can be used to obtain transfer functions from $\underline{\underline{A}}$ and $\underline{b}$ for use by CONOPT.

The handling-quality time histories, $y_i(t)$, are linear combinations of state variables,

$$\underline{y}(t) = \underline{\underline{G}}\underline{x}(t) \quad . \tag{4}$$

If all the $y_i(t)$ fall within their handling-quality time-history envelopes the model is appropriate for use with CONOPT.

The rigid-body equations of motion for symmetrical airplanes slightly disturbed from straight and level flight can be separated into two sets of four simultaneous first-order linear constant-coefficient ordinary differential equations [3]. One set describes the longitudinal motion, or motion in the plane of symmetry of a normally configured airplane. The other set describes the lateral motion. In each case,

$$d\underline{x} = \underline{\underline{A}}\underline{x} + \underline{b}u \quad , \tag{5}$$

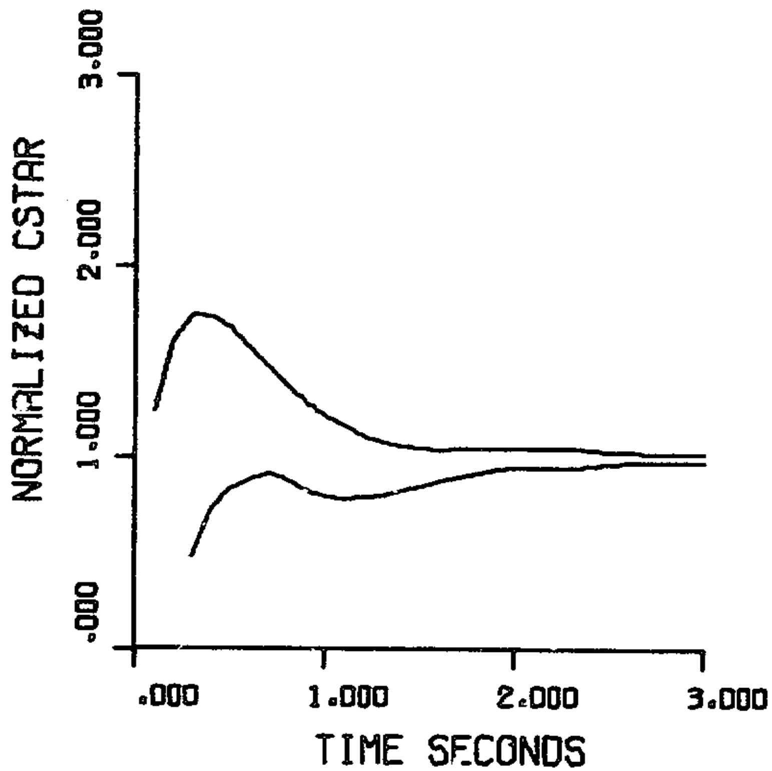
where $\underline{A}$ is 4 x 4 and $\underline{b}$ is 4 x 1 if there is only one input.

Only one longitudinal handling-quality criterion has been established: C_n^*. Three lateral handling-quality criteria have been proposed: D_n^*, normalized roll rate, and normalized sideslip. An envelope for the first derivative of each handling-quality time history has also been proposed [4]. These eight envelopes are plotted in Figures 1 through 4. These handling-quality time-history envelopes imply standard inputs. The C^* response results from a step change in elevator position, δ_ε, and the lateral handling-quality time responses result from a step change in aileron position, δ_a.

METHODS

An obvious procedure for obtaining models with handling-quality time histories which fall within specified envelopes is to:

1. Guess a model in terms of $\underline{A}$ and $\underline{b}$
2. Calculate $\underline{x}(t)$
3. Form $\underline{y}(t)$
4. Compare $\underline{y}(t)$ and $\dot{\underline{y}}(t)$ to the appropriate envelopes



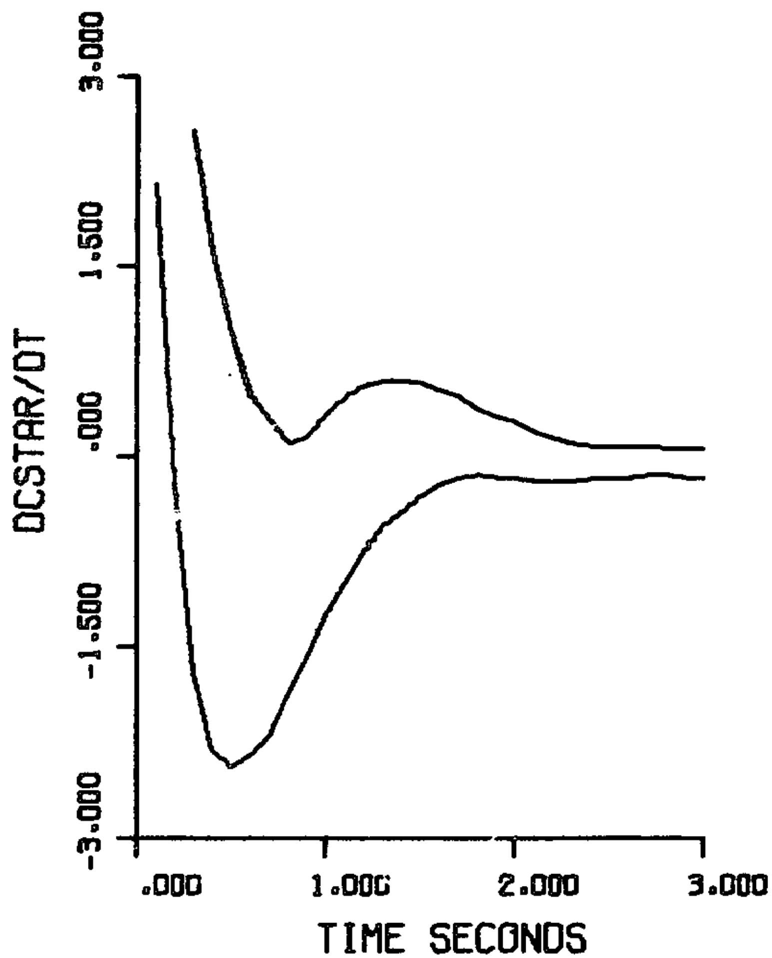


FIG.1 LONG. CRIT. ENVELOPES

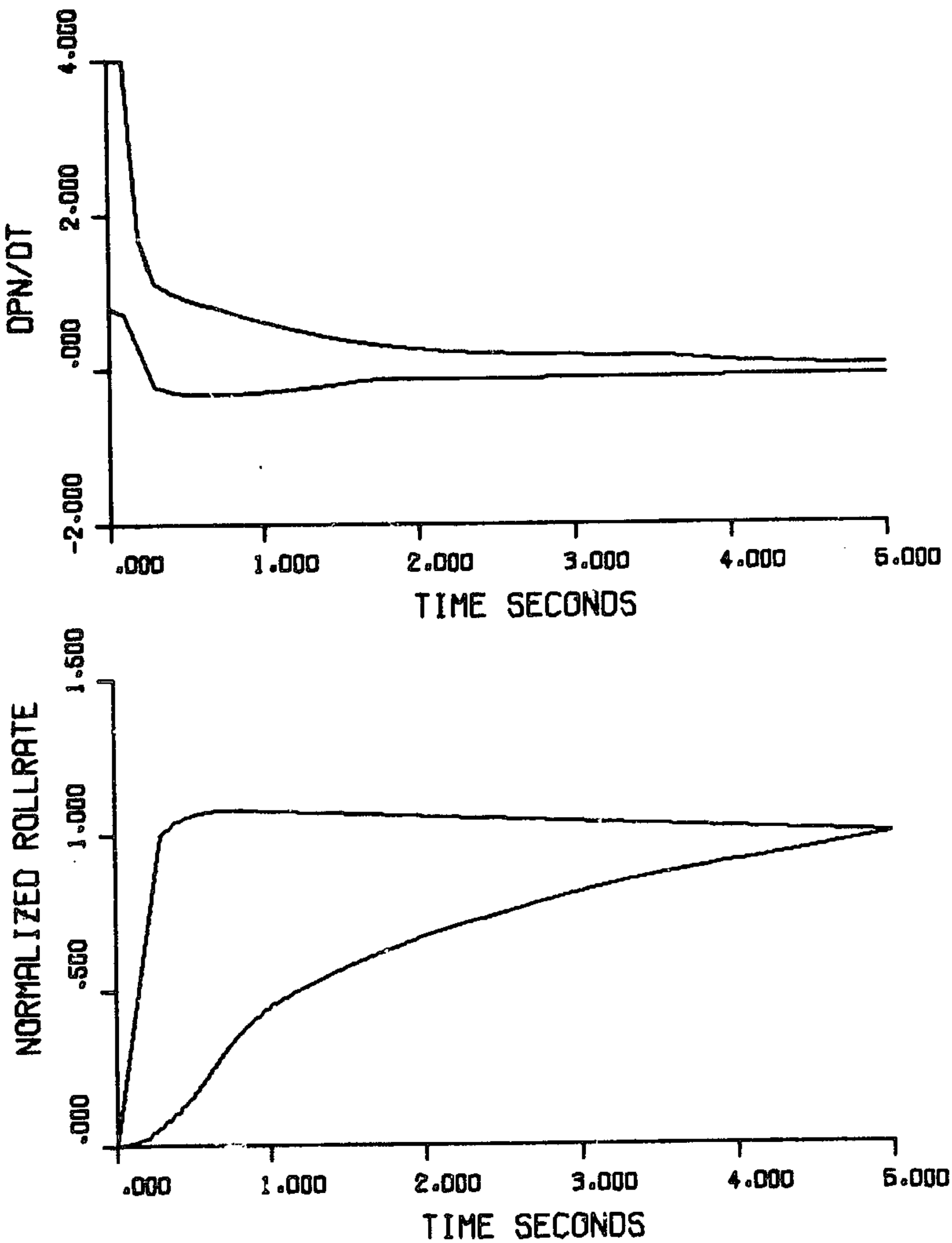


FIG.2 ROLLRATE ENVELOPES

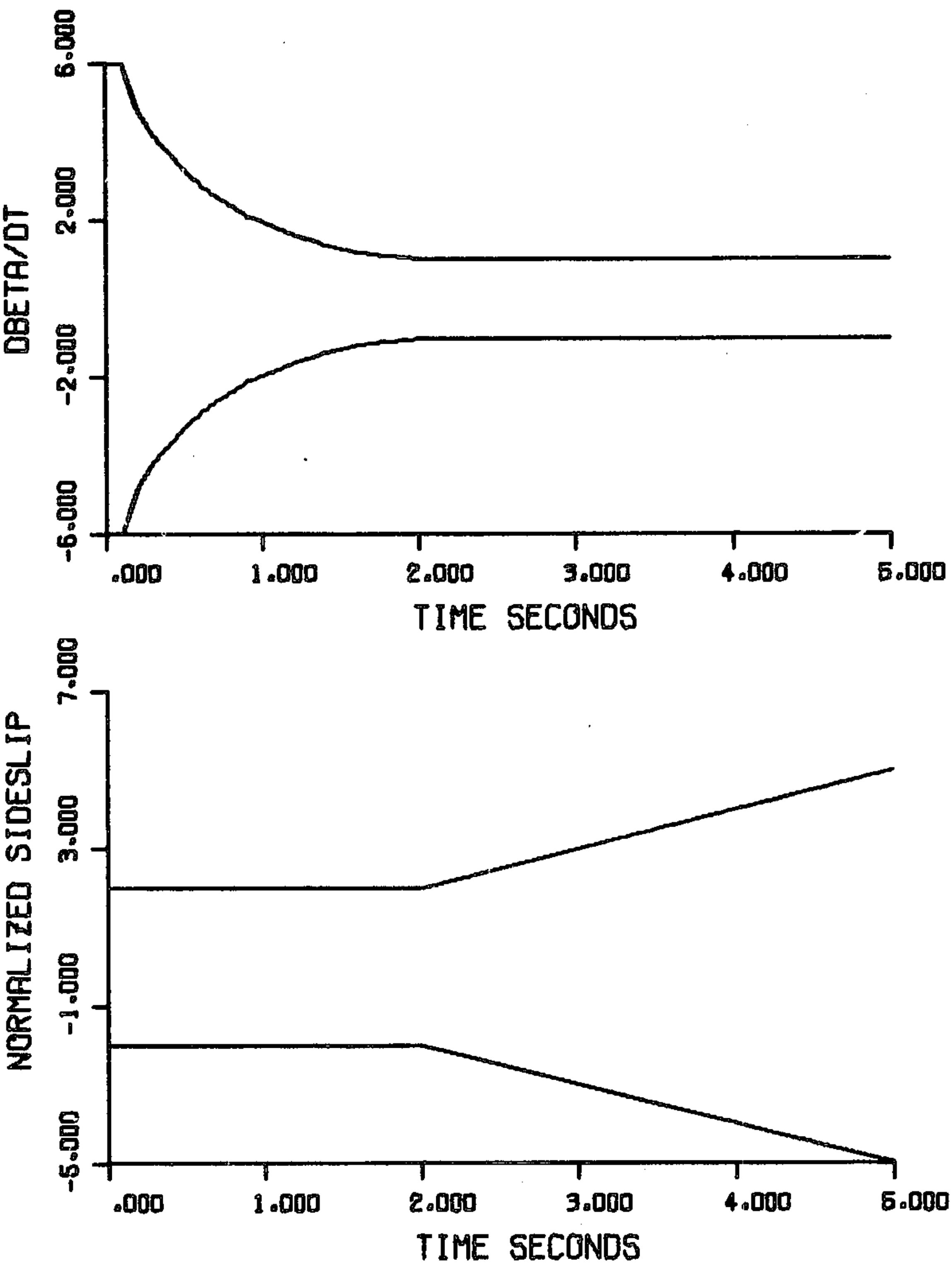


FIG.3 SIDESLIP ENVELOPES

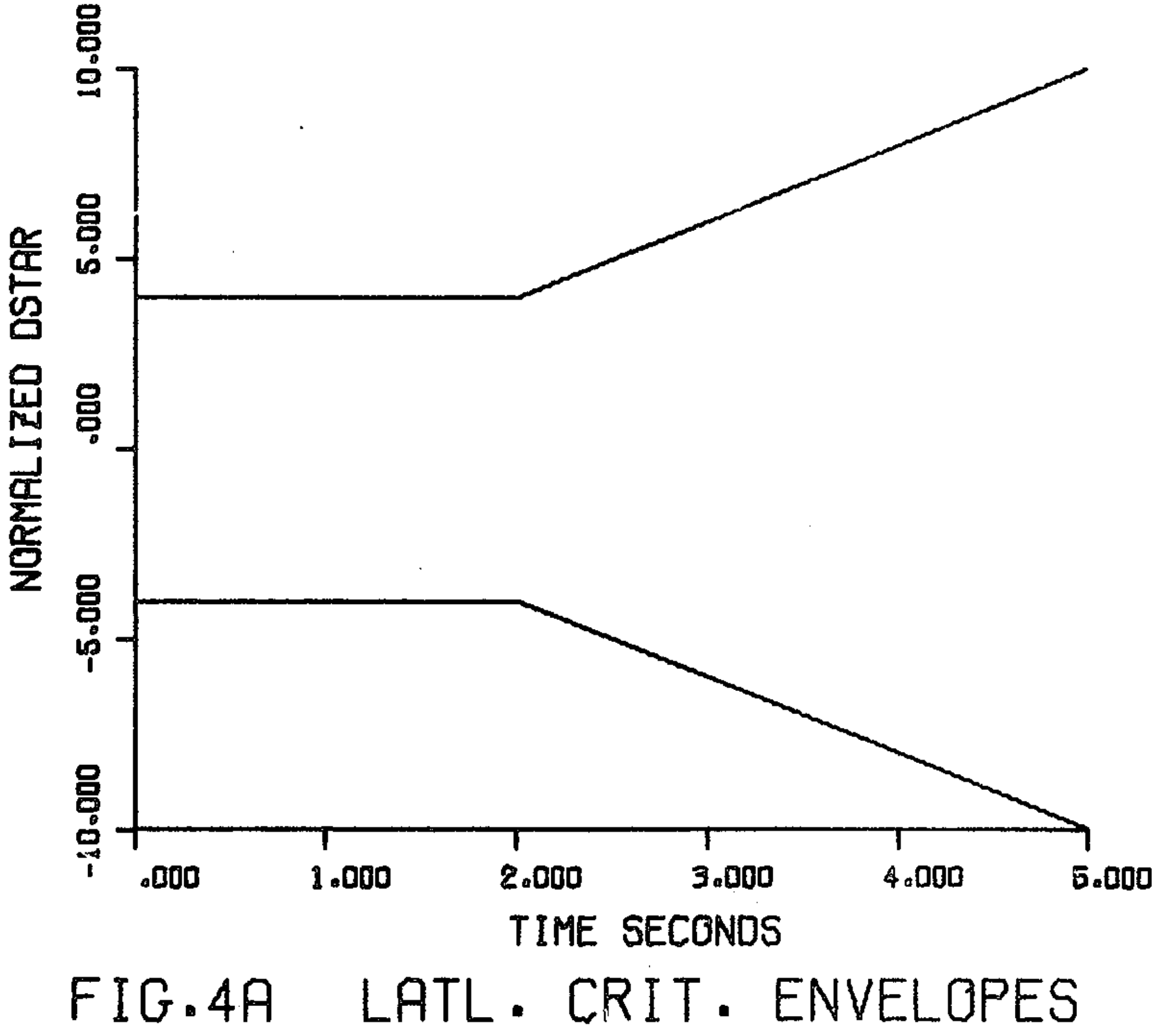
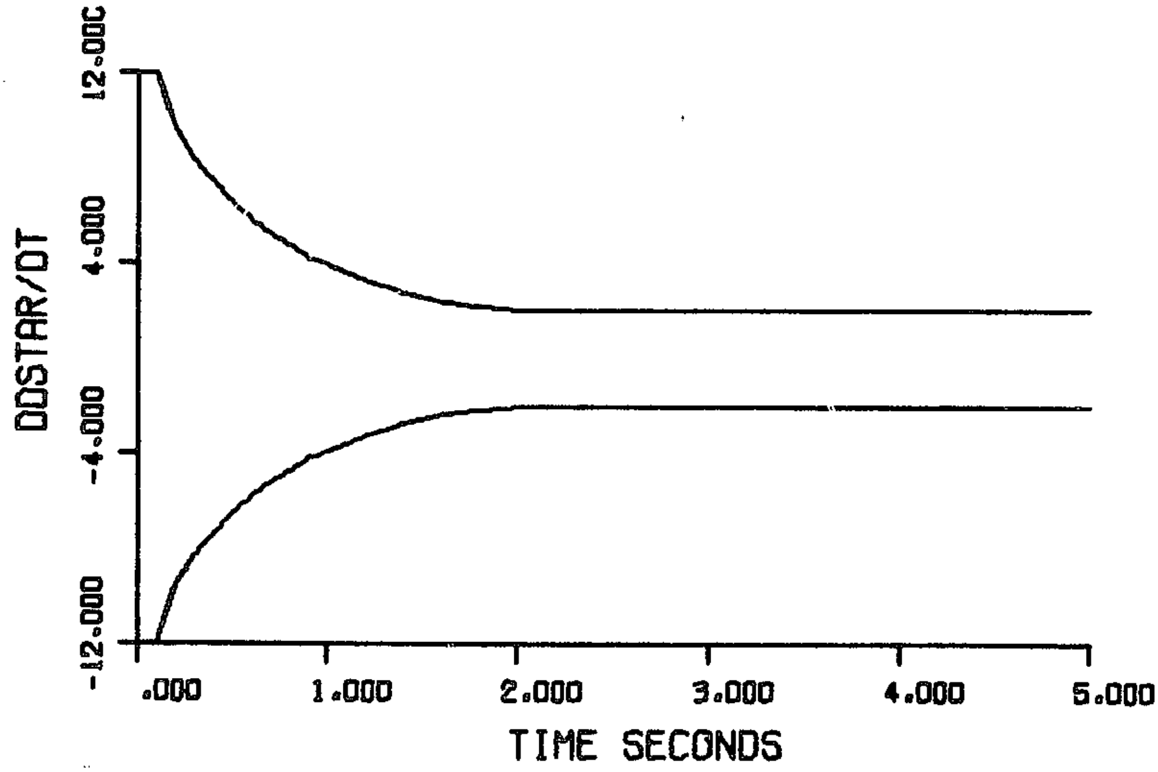


FIG.4A LATL. CRIT. ENVELOPES



12.00C
4.000
-4.000
-12.000
DDSTAR/DT
.000
1.000
2.000
3.000
4.000
5.000
TIME SECONDS

FIG. 4B

5. If no envelopes are violated, stop
6. Otherwise alter A and/or b according to some strategy and return to Step 2.

This "brute force" technique would consume large amounts of computer time and may not succeed. It would be necessary to define a cost functional which measures all excursions outside of the envelopes and which is suitable for use in a numerical optimization scheme. Next a connection between the elements a_{ij} and b_i and the cost functional would have to be established. Finally, there is no assurance that the attainable minima of this functional would be zero. The addition of hard and soft constraints arising from the loose bounds on the eigenvalues and specification of the pole-zero excess of the transfer functions complicates the numerical optimization problem. In the fourth-order lateral-motion case there would be: twelve variables to be manipulated, four soft constraint equations to be approximately satisfied, n hard constraint equations where n is the sum of the specified pole-zero excesses, and six envelopes to be matched.

One simplification which circumvents the need for a cost functional and optimization strategy is the use of direct search. If any solution which satisfies the constraints and the envelopes is as useful as any other, systematic or random direct search is easier to implement and no more wasteful of computer time than optimization. Random direct search has been used to obtain second-order longitudinal airplane transfer functions. In this simpler situation the $a_{ij} : i,j = 1,2$ are randomly selected. If the resulting eigenvalues fall within the region shown in Figure 5 [4], the $b_i : i = 1,2$ are randomly selected and C^* and $\dot{C}^*$ calculated. The ranges of the random choices are very

influential. Sample results are:

1. 558 random sets of a_{ij} yielded 22 acceptable pairs of real eigenvalues and six acceptable pairs of complex eigenvalues.
2. For $\underline{A}$ having acceptable real eigenvalues 118 random pairs of b_i were required to find one pair which yielded time-responses which match the C^* and $\dot{C}^*$ envelopes.
3. For $\underline{A}$ having acceptable complex eigenvalues 320 random pairs of b_i were required to find one pair which satisfied the envelopes.
4. If one envelope was matched (and the other violated) it was as likely to be the $\dot{C}^*$ envelope as the C^* envelope.

The application of direct search to the fourth-order problem is currently being investigated. This is potentially useful for the development of models which satisfy the longitudinal criteria, C^* and $\dot{C}^*$, but less so for the lateral case. In order to develop lateral motion models and obviate the use of iterative numerical techniques the problem must be reposed.

If one begins with specific time responses which fall within the envelopes and seeks a model whose output responses closely resemble the specified handling-quality time histories the problem no longer requires iteration. If the eigenvalues are specified or obtained from the input responses and then frozen and if all elements of $\underline{A}$ and $\underline{b}$ are unspecified rather than just those elements which are normally nonzero for most airplanes, the problem is well-posed. This procedure has three disadvantages:

1. The partial degrees of freedom represented by loose bounds on

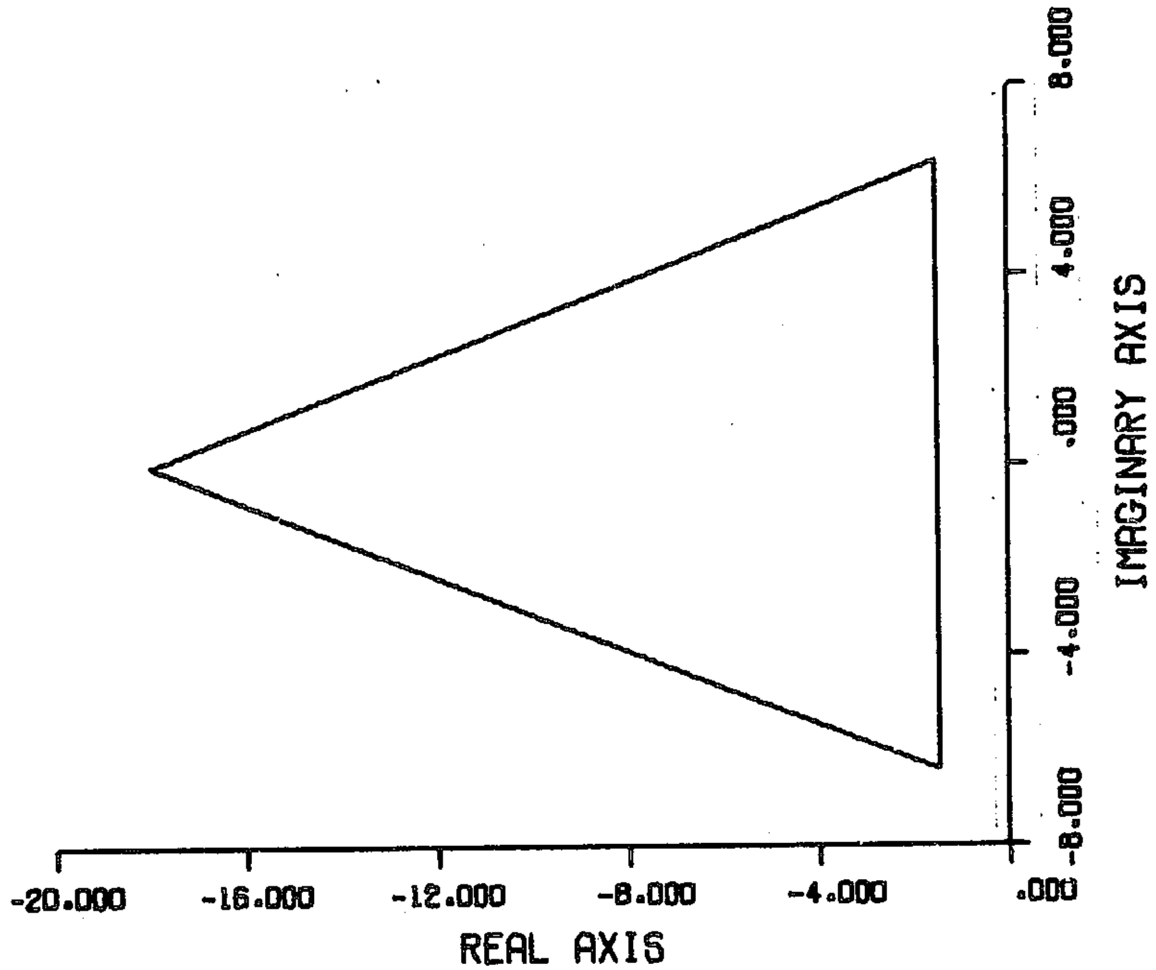


FIG.5 ACCEPTABLE EIGENVALUES

on the eigenvalues are lost.

2. The envelopes are not directly used and may be violated by the results.

3. The resulting $\underline{A}$ is not constrained to have the zero elements and kinematic elements usually found in such models of airplanes.

Experience with lateral-motion examples has shown that $\underline{A}$ may well have obviously unrealistic elements and still produce realistic transfer functions. This is sufficient for present purposes.

The reposed problem is the following: given $\hat{\underline{y}}(t)$, $\underline{h}(a_{ij})$ and $\underline{G}(a_{ij})$, find $\underline{A}$ and $\underline{b}$ such that if

$$d\underline{x} = \underline{A}\underline{x} + \underline{b}\delta_a \quad , \tag{6}$$

and if

$$\underline{y} = \underline{G}\underline{x} + \underline{h}\delta_a \quad , \tag{7}$$

then

$$\underline{y} \approx \hat{\underline{y}} \text{ in some best sense,}$$

with

$$\underline{\Lambda} = \hat{\underline{\Lambda}} \tag{8}$$

Note that the elements of $\underline{G}$ and $\underline{h}$ are known functions of the unknown elements of $\underline{A}$ and that $\underline{G}$ is normally not square, there being fewer elements in $\underline{y}$ than in $\underline{x}$.

The first step is to obtain an analytical representation of the input time histories which one specifies in the form of a table of discrete values uniformly spaced in time. If the model is to be a fourth-order constant-

coefficient linear ordinary differential equation, its solution must have the form:

$$y_i(t) = c_{i1}e^{\lambda_1 t} + c_{i2}e^{\lambda_2 t} + c_{i3}e^{\lambda_3 t} + c_{i4}e^{\lambda_4 t} + c_i \quad . \tag{9}$$

These coefficients and eigenvalues, c_{ij} and λ_j, are obtained by least-squared-error fitting the above form to the specified discrete values of $\hat{y}_i$. This is a two-step process. First the λ_j are calculated. The calculation is based upon first and second differences between adjacent $\hat{y}_i$ values and is strongly influenced by their precision. If the values are represented by three significant figures the resulting eigenvalues are drastically incorrect even when the data are contrived and there should be an exact solution. As the number of significant figures is increased the calculated eigenvalues more closely resemble the correct values. However, if data obtained from sketched time responses or imprecise tables are to be accommodated, it is impractical to calculate eigenvalues from such data. In such cases, the eigenvalues which the model is to have must be specified. In either case, the c_{ij} are straightforwardly calculated and the resulting fits usually quite good. Thus one can represent the input time histories, $\hat{y}(t)$, as

$$\underline{\hat{y}} \approx \underline{y} = \underline{\underline{C}}e^{\underline{\lambda}t} + \underline{c} \tag{10}$$

where the elements of $\underline{c}$ are the c_i .

An example set of discrete input data points and their fitted representations is shown in Figures 6 through 8. Reasonable responses were sketched onto graphs of the lateral handling-quality envelopes and then converted to tabular form by estimating the values at half-second intervals. The eigenvalues used in the curve-fitting process were specified. The values used were:

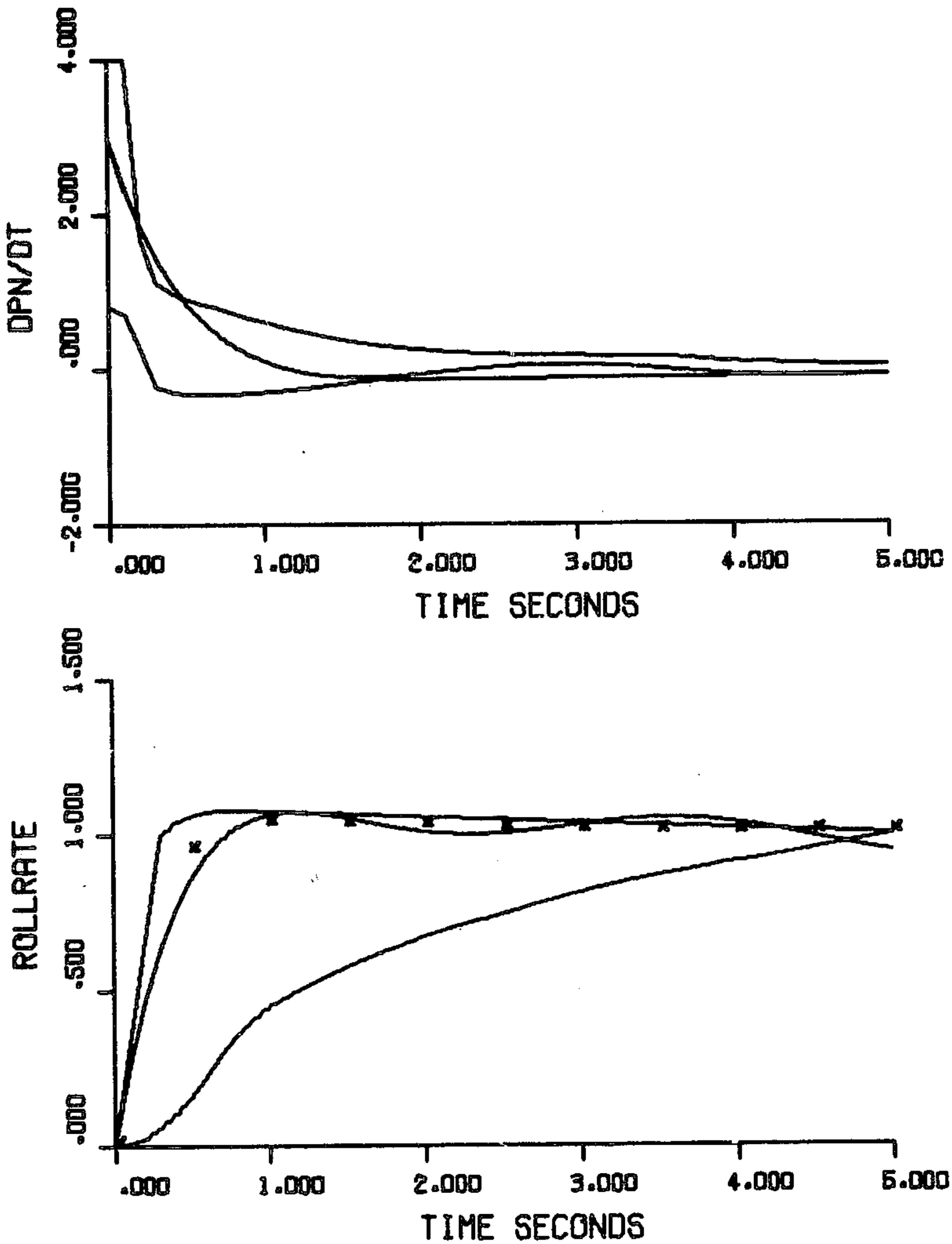


FIG.6 FITTED ROLLRATE RESPONSE

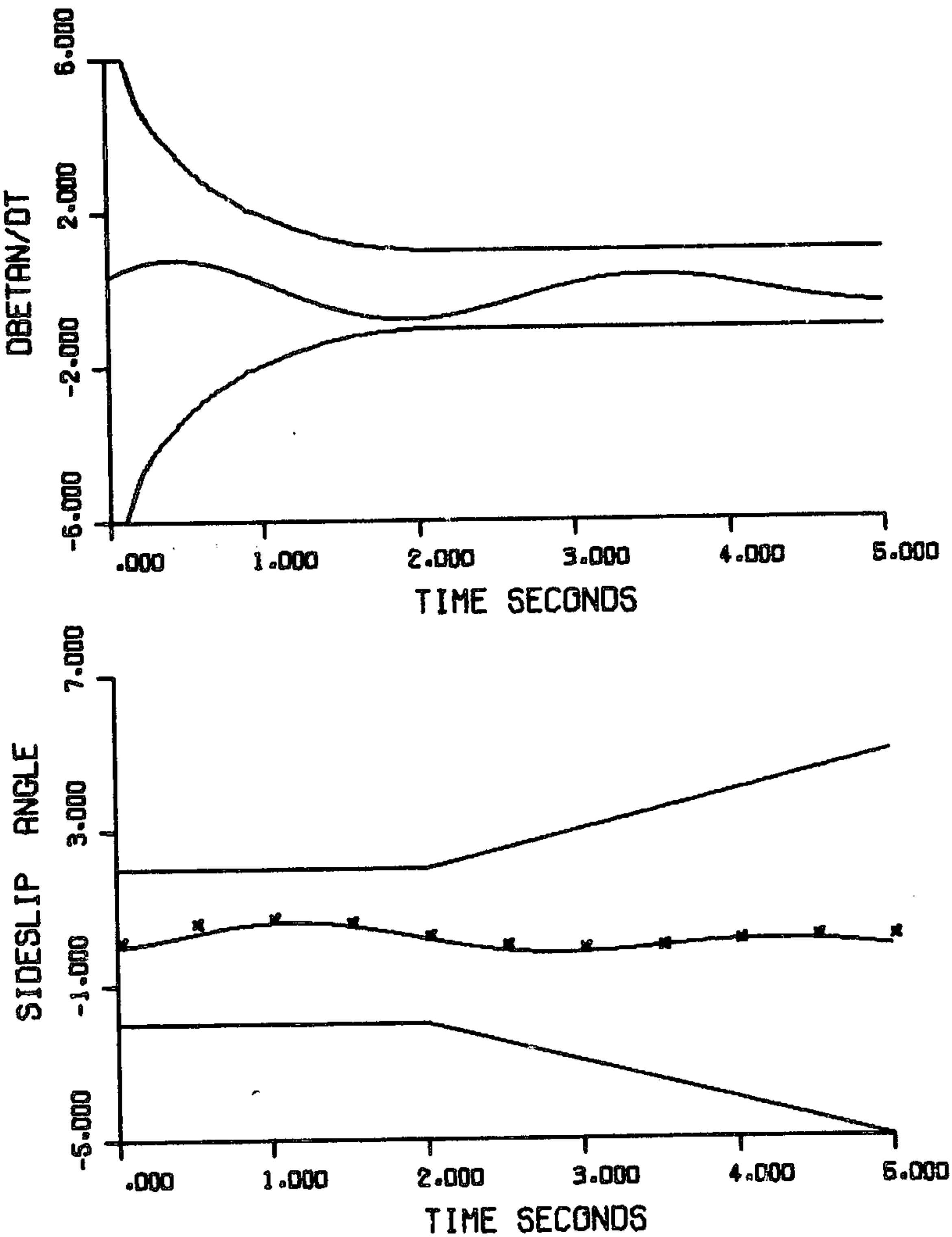


FIG.7 FITTED SIDESLIP RESPONSE

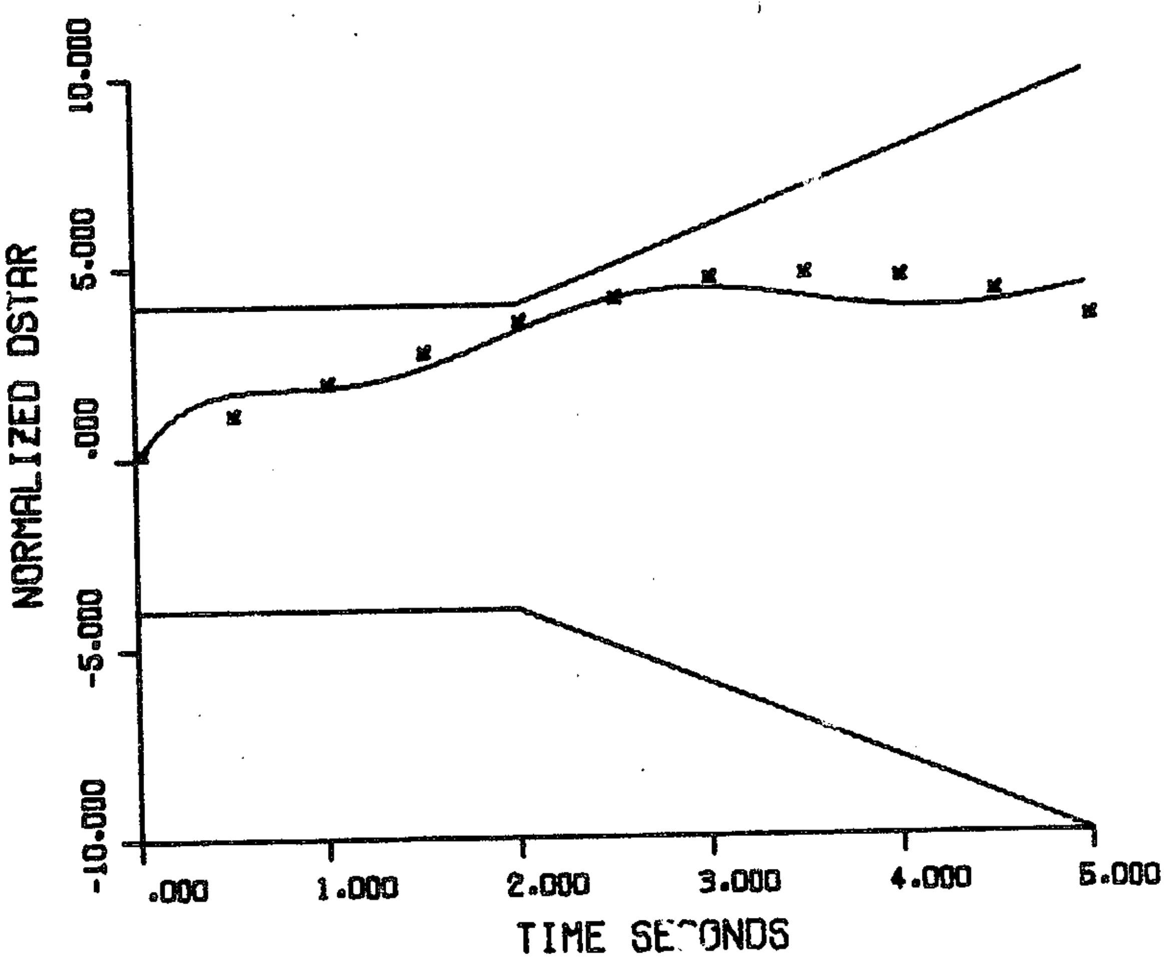
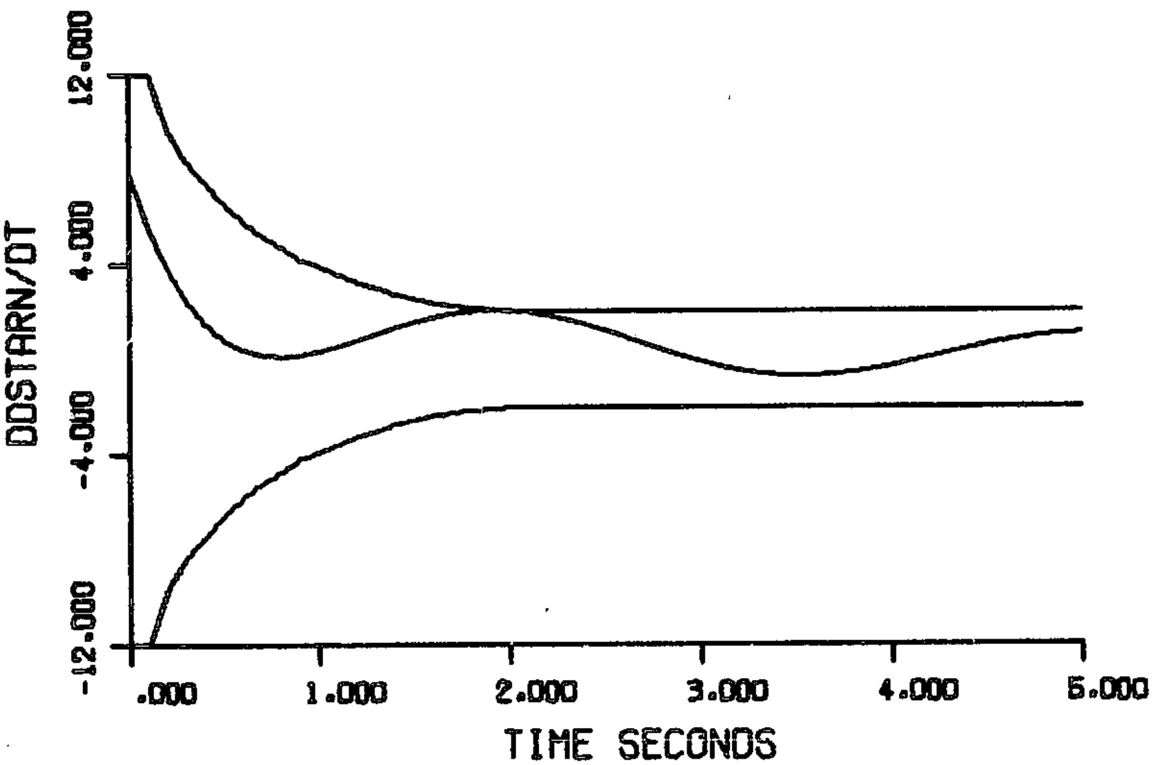


FIG.8A FITTED DSTAR RESPONSE



12.000
4.000
-4.000
-12.000
DDSTARN/DT
.000
1.000
2.000
3.000
4.000
5.000
TIME SECONDS

FIG. 8B

$$\lambda_1 = -2.4$$

$$\lambda_2 = -.003$$

$$\lambda_3 = -.25 - j2.0$$

$$\lambda_4 = -.25 + j2.0$$

Once the specified discrete values of the $\hat{y}_i$ are approximately represented

$$\underline{y} = \underline{\underline{C}}\underline{e}^{\underline{\lambda}t} + \underline{c}$$

This equation can be Laplace transformed into

$$\underline{Y}(s) = \frac{\underline{N}(s)}{D(s)} \quad . \tag{11}$$

Assuming a unit-step input, the plant and output equations can also be Laplace transformed into

$$s\underline{X}(s) = \underline{\underline{A}}\underline{X}(s) + \underline{b}/s$$

$$\underline{Y}(s) = \underline{\underline{G}}\underline{X}(s) + \underline{h}/s$$

which are manipulated to obtain

$$\underline{X}(s) = (s\underline{I} - \underline{A})^{-1}\underline{b}/s = \underline{\phi}(s)\underline{b}/s$$

$$\underline{Y}(s) = \underline{\underline{G}}\underline{\phi}(s)\underline{b}/s + \underline{h}/s \quad . \tag{12}$$

If equation 12 is rearranged to match the form of equation 11, the right-hand sides of equations 11 and 12 can be equated, coefficient by coefficient, to produce n(m+1) simultaneous algebraic equations where n is the order of the plant equation, equation 6, and m is the number of elements in the output vector, equation 7. For low-order models this is a satisfactory technique.

The algebraic equations are of degree $\leq n$. In the fourth-order lateral-motion example there are sixteen nonlinear equations containing, at worst, quadruple-cross-product terms. A limited amount of experience with these equations has shown them to be numerically sensitive. An effort is being made to develop a computer program which will solve the fourth-order example equations with various constraints on the elements and eigenvalues of $\underline{A}$. While this may prove to be the most satisfactory approach to the problem, the numerical uncertainties are sufficient to motivate the development of an alternative technique which is computationally simpler.

A less numerically-sensitive approach can be developed by combining equations 7 and 10.

$$\underline{y} = \underline{G}\underline{x} + \underline{h}\delta_a$$

$$\underline{x} = \underline{G}^{-1}\underline{y} - \underline{G}^{-1}\underline{h}\delta_a$$

$$\underline{x} = \underline{G}^{-1}\underline{C}\underline{e}^{\lambda t} + \underline{G}^{-1}\underline{c} - \underline{G}^{-1}\underline{h}\delta_a$$

$$d\underline{x} = \underline{G}^{-1}\underline{C}\underline{\Lambda}\underline{e}^{\lambda t} - \underline{G}^{-1}\underline{h}d\delta_a$$

If these expressions are substituted into equation 6 one obtains

$$\underline{G}^{-1}\underline{C}\underline{\Lambda}\underline{e}^{\lambda t} - \underline{G}^{-1}\underline{h}d\delta_a = \underline{A}\underline{G}^{-1}\underline{C}\underline{e}^{\lambda t} + \underline{A}\underline{G}^{-1}\underline{c} - \underline{A}\underline{G}^{-1}\underline{h}\delta_a + \underline{b}\delta_a$$

or

$$(\underline{G}^{-1}\underline{C}\underline{\Lambda} - \underline{A}\underline{G}^{-1}\underline{C})\underline{e}^{\lambda t} = \underline{A}\underline{G}^{-1}\underline{c} + \underline{G}^{-1}\underline{h}d\delta_a + (\underline{b} - \underline{A}\underline{G}^{-1}\underline{h})\delta_a \quad .$$

For this expression to be true

$$\underline{G}^{-1}\underline{C}\underline{\Lambda} = \underline{A}\underline{G}^{-1}\underline{C} \tag{13a}$$

and

$$(\underline{A}\underline{G}^{-1}\underline{h} - \underline{b})\delta_a = \underline{A}\underline{G}^{-1}\underline{c} \qquad (13b)$$

If $\underline{T} = \underline{G}^{-1}\underline{C}$, then

$$\underline{A} = \underline{T}\underline{\Lambda}\underline{T}^{-1} \qquad (14)$$

and

$$\underline{b} = \underline{A}\underline{G}^{-1}(\underline{h} - \underline{c}) \quad . \qquad (15)$$

It is assumed that $\delta_a(t) \equiv 1$ and $\underline{G}^{-1}\underline{h}d\delta_a$ is neglected at this point but must be introduced as an initial condition vector when computing model time responses. A model obtained from equations 14 and 15 will have the specified eigenvalues and match the analytical representations of the input data exactly.

Unfortunately the above results assume the existence of $\underline{G}^{-1}$ whereas, in general, $\underline{G}$ is not square. One can substitute the right pseudoinverse and obtain similar results except that $\underline{T}$ will be singular and the model eigenvalues are no longer equal to the specified eigenvalues since $\underline{A}$ and $\underline{\Lambda}$ are no longer similar. If $\underline{G}$ is square and nonsingular, equations 14 and 15 hold. There are two cases of practical interest in which $\underline{G}$ is nonsingular. First, one can specify $n = m$, that is, let the number of time histories establish the order of the model. Secondly, it may be possible to adjoin created pseudo-time-histories to $\underline{y}$ until $m = n$. One must have the means to create these pseudo-time-histories. They must be linear combinations of the model state variables and the resulting distribution matrix must be nonsingular.

In the lateral-direction, small-motion example, the state variables are roll rate, yaw rate, sideslip angle and roll angle. The handling-quality indicators are normalized roll rate, normalized sideslip angle and normalized D^*.

The normalized roll-rate input data can be integrated to produce normalized roll-angle pseudodata. For the example, this was accomplished by exactly fitting a tenth-order polynomial to the eleven discrete roll-rate data points shown in Figure 6. This polynomial was integrated and evaluated at the same values of time to generate the discrete pseudodata points shown in Figure 9. The continuous curve in Figure 9 results from fitting an equation of the form of equation 9 to the discrete pseudodata points just as is done for the normal data. This allows the expansion of $\underline{y}$ to $\tilde{\underline{y}}$ and $\underline{G}$ to $\tilde{\underline{G}}$ and assures the non-singularity of $\tilde{\underline{G}}$. This, in turn, allows calculation of $\underline{A}$ and $\underline{b}$ using equations 14 and 15. The same straightforward opportunity to augment $\underline{y}$ until $\tilde{\underline{G}}$ is square and nonsingular does not exist in the longitudinal small-motion case, although a designer could impose specifications on the motion, in addition to the Boeing C^* criterion, and thereby create a nonsingular $\tilde{\underline{G}}$.

COMPUTATIONAL PROCEDURES

D^* is a weighted combination of aircraft sideslip which is considered the principal low-speed handling-quality parameter and lateral acceleration at the pilot station which is the principal motion cue parameter at high speeds [4]. In terms of a stability-axis system representation of small perturbations about straight and level flight for a normally configured aircraft with the pilot station on the longitudinal principal axis the expression for D^* is

$$D^*(t) = V(\mathring{\beta} + r) + \ell\mathring{r} + c_3 q_{co}\beta \tag{16}$$

where c_3 is a dimensional constant defined in Table 1. In terms of the

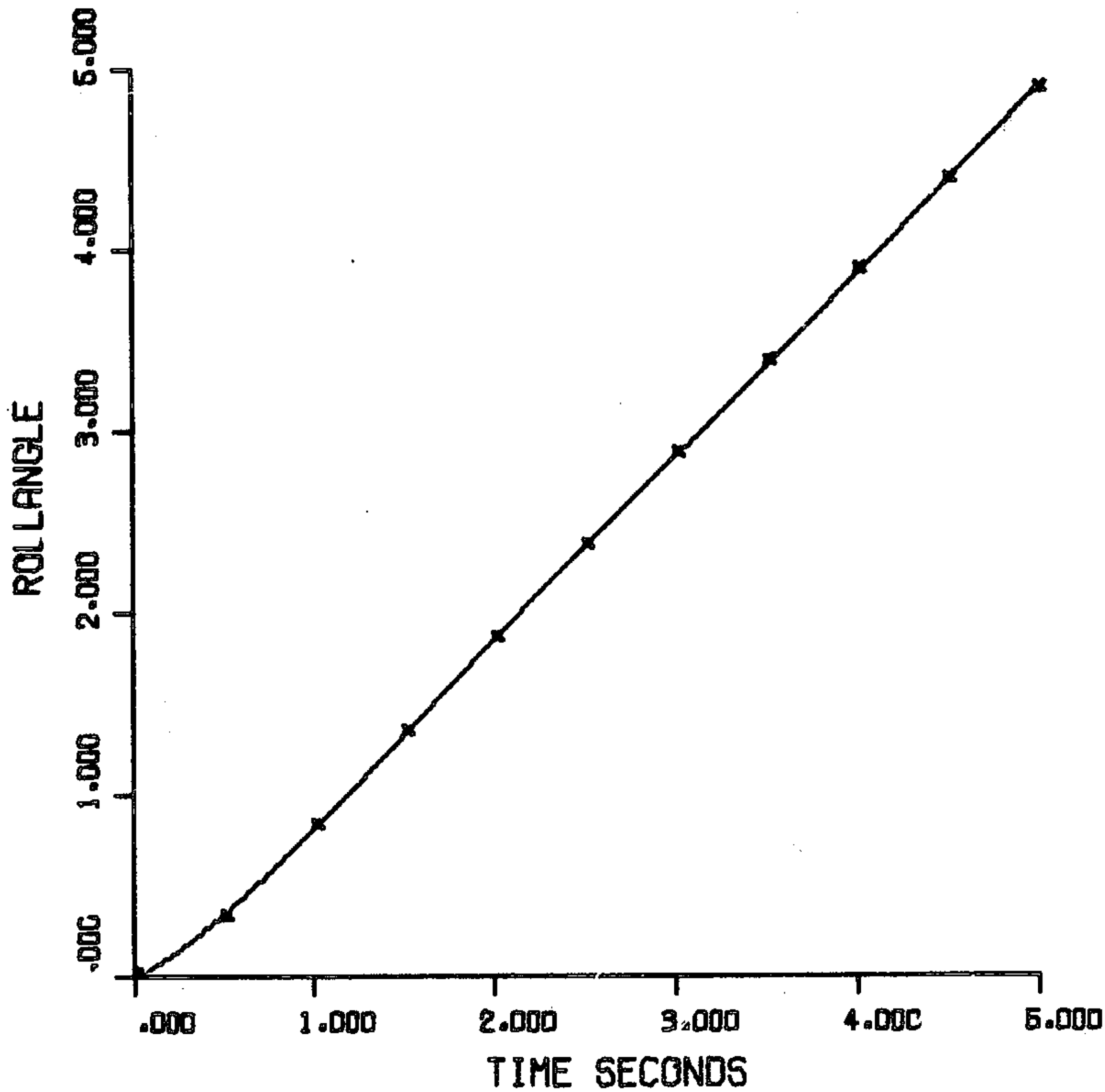


FIG.9 ROLLANGLE PSEUDODATA

elements $\underline{A}$ and $\underline{b}$ one obtains

$$D_p = Va_{31} + \ell a_{21}$$

$$D_r = Va_{32} + \ell a_{22} + V$$

$$D_\beta = Va_{33} + \ell a_{23} + c_3 q_{co}$$

$$D_\phi = Va_{34} + \ell a_{24}$$

$$D_{\delta_a} = Vb_3 + \ell b_1$$

and

$$D^*(t) = D_p p(t) + D_r r(t) + D_\beta \beta(t) + D_\phi \phi(t) + D_{\delta_a} \delta_a \quad . \tag{18}$$

Unit Correction Factors for D^* Equation

D^*	β	C_3	
		Value	Units
g's	rad	-9.91×10^{-3}	$\left(\frac{g's - ft^2}{lb}\right)$
	deg	-1.73×10^{-4}	$\left(\frac{g's - ft^2}{lb - deg}\right)$
ft/sec^2	rad	-3.19×10^{-1}	$\left(\frac{ft^3}{lb\text{-}sec^2}\right)$
	deg	-5.57×10^{-3}	$\left(\frac{ft^3}{lb\text{-}sec^2\text{-}deg}\right)$

Units for crossover dynamic pressure, q_{co}; lb/ft^2

Table 1 [4]

Note that this assumes that the plant matrix is

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & & \\ \vdots & & \end{bmatrix}$$

With no constraints on the elements a_{ij}. Thus $\underline{A}$ is specifically not constrained to be of the form

$$\underline{A} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ \alpha_o & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix}, \tag{19}$$

similarly

$$\underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad \text{and not} \quad \begin{bmatrix} L_{\delta_a} \\ N_{\delta_a} \\ Y_{\delta_a} \\ 0 \end{bmatrix}.$$

Thus the output distribution matrix $\underline{G}$ for the lateral motion case is of the form

$$\underline{G} = \begin{bmatrix} 1/p_{ss} & 0 & 0 & 0 \\ 0 & 0 & 1/\beta_{ss} & 0 \\ D_p/D_{ss} & D_r/D_{ss} & D_\beta/D_{ss} & D_\phi/D_{ss} \end{bmatrix} \tag{20}$$

where

$$y_1(t) = p(t)/p_{ss} = p_n(t)$$

$$y_2(t) = \beta(t)/\beta_{ss} = \beta_n(t)$$

$$y_3(t) = D^*(t)/D_{ss} = D_n^*(t) \tag{21}$$

and

$$\underline{h} = \begin{bmatrix} 0 \\ 0 \\ D_{\delta_a}/D_{ss} \end{bmatrix} \tag{22}$$

The augmented output distribution matrix is

$$\tilde{\underline{G}} = \begin{bmatrix} 1/p_{ss} & 0 & 0 & 0 \\ 0 & 0 & 1/\beta_{ss} & 0 \\ 0 & 0 & 0 & 1/p_{ss} \\ D_p/D_{ss} & D_r/D_{ss} & D_\beta/D_{ss} & D_\phi/D_{ss} \end{bmatrix} \tag{23}$$

which is nonsingular if $D_r \neq 0$. The augmented output vector, $\tilde{\underline{y}}(t)$, is

$$\begin{aligned} \tilde{y}_1(t) &= p_n(t) \\ \tilde{y}_2(t) &= \beta_n(t) \\ \tilde{y}_3(t) &= \int p_n(t)dt = \phi_n(t) \\ \tilde{y}_4(t) &= D_n^*(t) \end{aligned} \tag{24}$$

Since n^2 elements of $\underline{A}$ and the n elements of $\underline{b}$ are to be established equation 14 is equivalent to n^2 scalar equations with n^2 unknowns and equation 15 is equivalent to n scalar equations with n unknowns. This is not significantly different from the Laplace transform method in which one obtains n(m+1) algebraic equations without having had to create psuedodata. However, the degree

of the simultaneous equations to be solved in the Laplace transform method is n whereas the equations arising from the pseudodata method contain cross-product terms at worst. This dramatically increases the liklihood of obtaining solutions by iterative numerical means. For the fourth-order lateral motion example, there are sixteen equations to be solved for the a_{ij}. A Newton-Euler procedure [5] was employed in which an initial estimate of the solution vector $\underline{a}$ is iteratively improved. If equation 14 is rewritten

$$\underline{f}(\underline{a}) = \underline{0} \tag{25}$$

and $\underline{P}(\underline{a})$ is the Jacobian matrix associated with the equation 25 then

$$\underline{a}_{k+1} = \underline{a}_k - \underline{P}^{-1}(\underline{a}_k)\underline{f}(\underline{a}_k) \tag{26}$$

This simple procedure has proved to be so successful that $\underline{A} = \underline{I}$ can be used as the starting point and every element of $(\underline{a}_{k+1} - \underline{a}_k)$ is reduced to 10^{-5} in three iterations typically. The elements of $\underline{b}$ are obtained from equation 15 without iteration.

Before $\underline{A}$ and $\underline{b}$ can be calculated, equations of the form of equation 10 must be fitted to the input data. The resulting coefficient arrays $\underline{C}$ and $\underline{c}$ appear in equations 14 and 15. Note that $\underline{T} = \underline{\tilde{G}}^{-1}\underline{C}$ in the pseudodata method.

A set of eigenvalues can be obtained from each discretized input time history. If

$$e_z = y_i(z\Delta t) - \hat{y}_i(z\Delta t)$$

and

$$\mu_j = e^{\lambda_j \Delta t}$$

then

$$e_z = c_{i1}\mu_1^z + c_{i2}\mu_2^2 + c_{i3}\mu_3^z + c_{i4}\mu_4^z + c_i - \hat{y}_i(z\Delta t) \tag{27}$$

where

$$z = 1,2,\ldots(\text{number of discrete values } \hat{y}_i(z\Delta t)).$$

The c_{ij} and c_i can be eliminated by linearly combining the e_z. One obtains a linear set of simultaneous algebraic equations

$$\underline{D}'\underline{\mu} = \underline{d}' \tag{28}$$

where the elements of D' are first differences of the table of $\hat{y}_i(z\Delta t)$ values and the elements of $\underline{d}'$ are second differences. Unfortunately, if the $\hat{y}_i(z\Delta t)$ values are obtained from time-response sketches or are imprecise for any reason the μ_j calculated from equation 28 are uselsss. Frequently the μ_j obtained in such cases have negative values from which no λ_j can be calculated.

In lieu of eiganvalues obtained from the input data, it has been necessary , in all practical calculations, to use specified eigenvalues. In such cases the calculation of $\underline{C}$ is based on minimizing

$$E = \sum_{z=1}^{11} e_z^2$$

where the lateral handling-quality time histories typically span five seconds and are discretized by taking values every half second for a total of eleven values per time history. The initial value of $\underline{\tilde{y}}(t)$ is forced to be zero by

$$c_i - \sum_{j=1}^{4} c_{ij} \quad . \tag{29}$$

As a check on the entire computational process once an $\underline{A}$ and $\underline{b}$ have been

calculated, $\underline{G}$ and $\underline{h}$ can be calculated, $\underline{x}(t)$obtained by integration and $\underline{y}(t)$ calculated. This $\underline{y}(t)$, obtained by integration, is plotted with the fitted $\underline{y}(t)$. They should be identical. The integration method is described in Takahashi, et. al., page 103 [6]. The solution matrix is represented by a series expansion containing p terms. The number of terms is specified by a recipe attributed to Paynter

$$\frac{1}{p!} (nq)^p e^{nq} \approx 0.001 \tag{30}$$

where n is the size of the $\underline{A}$ matrix and q is the largest element of $\underline{A}\Delta t$.

The example fitted and integrated handling-quality time histories are plotted in Figures 10, 11, and 12.

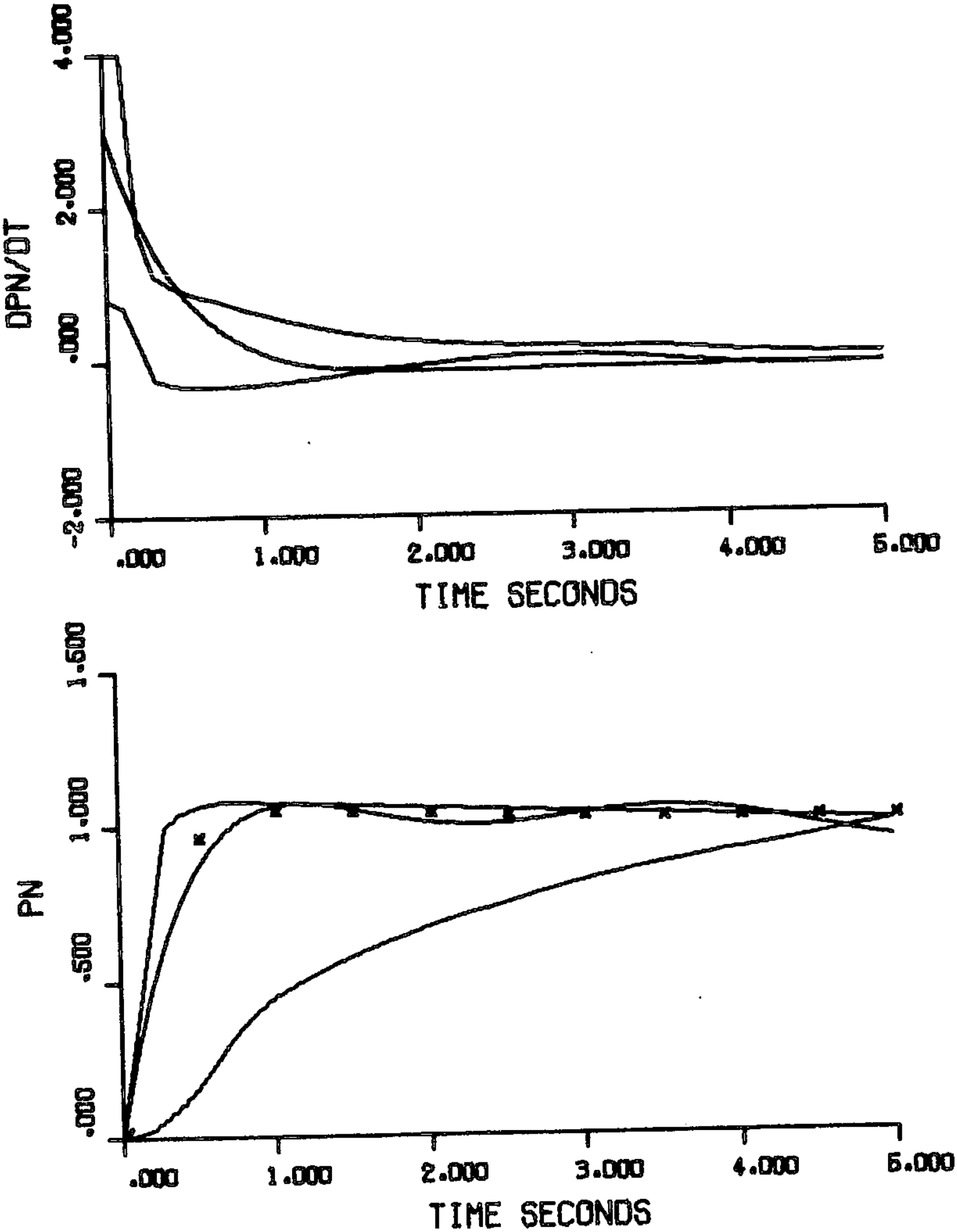


FIG.10 ROLLRATE

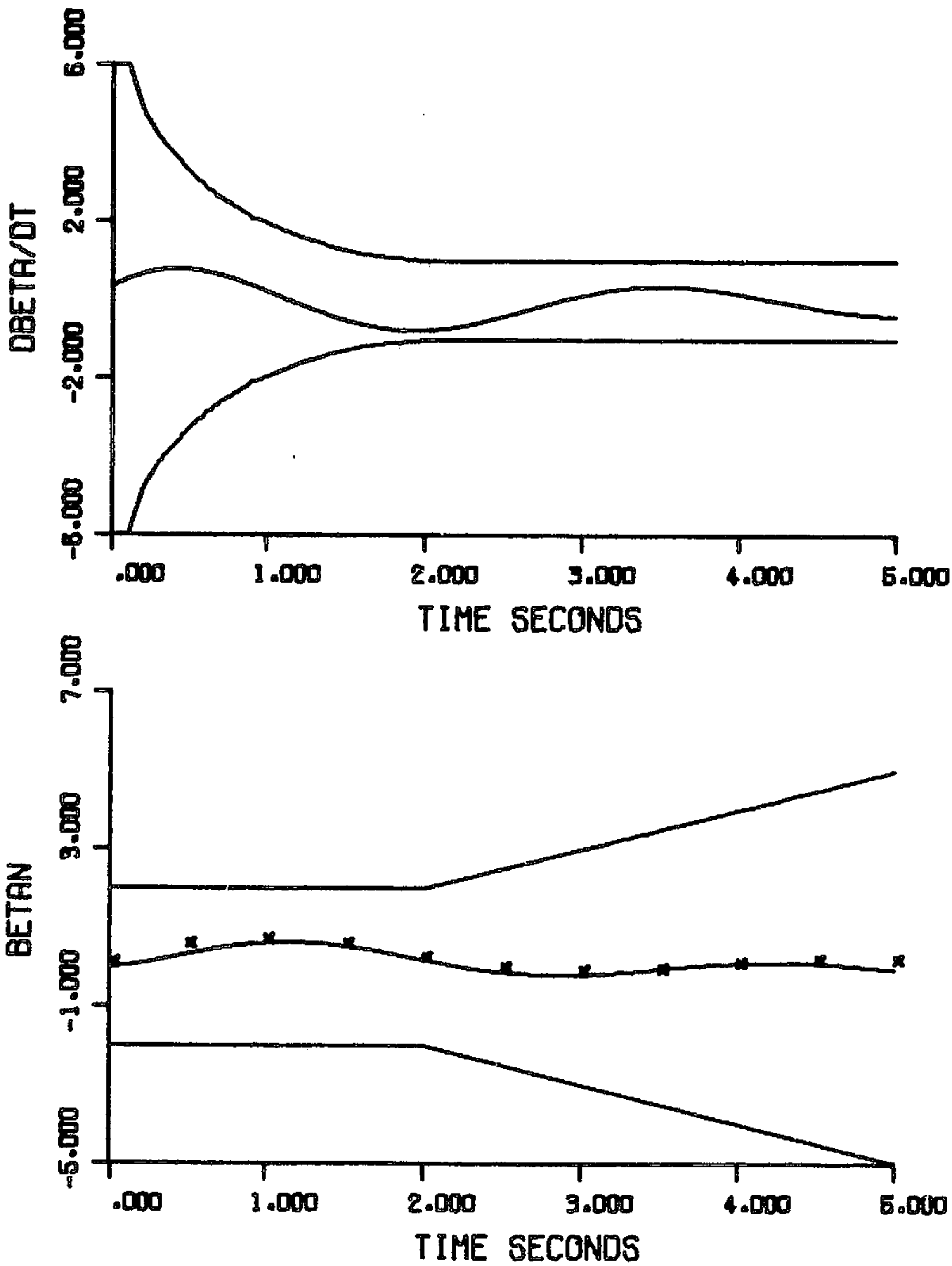


FIG.11 SIDESLIP

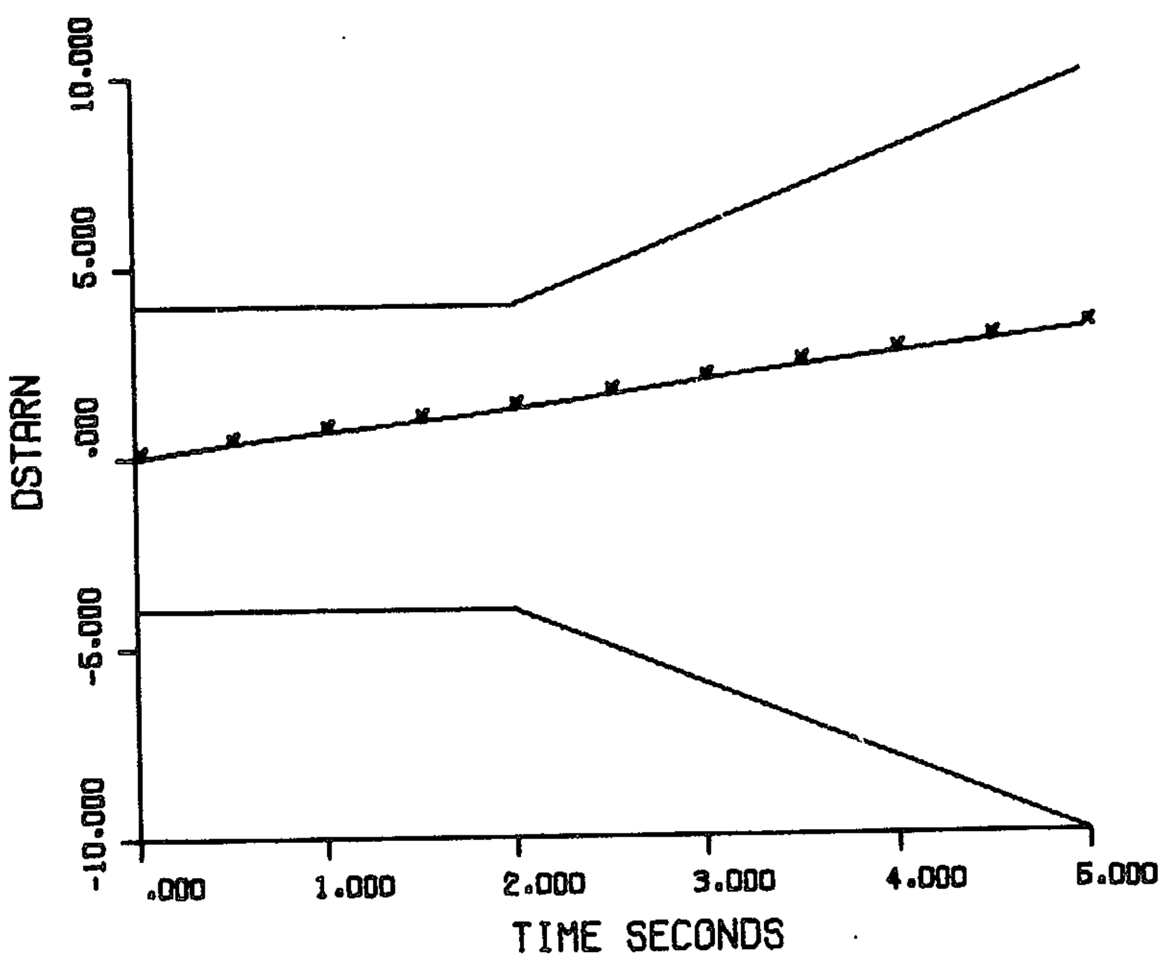


FIG.12 A LATERAL CRITERION

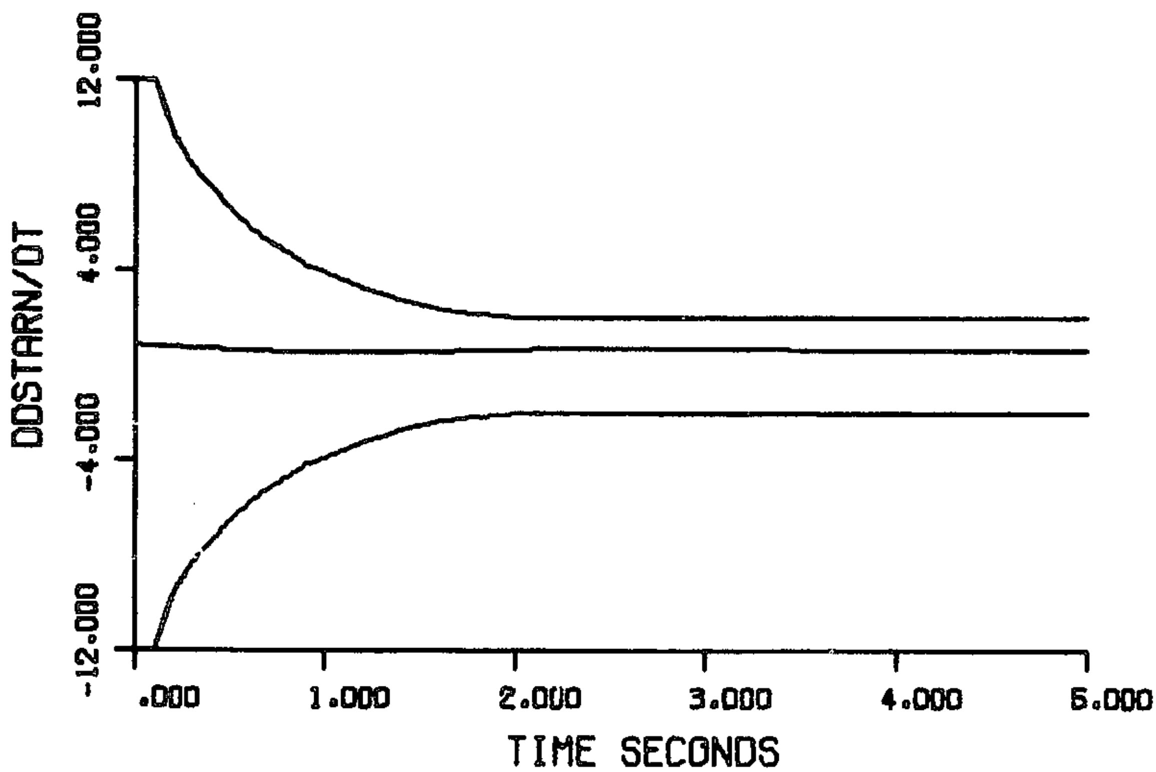

DDSTARN/DT
12.000
4.000
-4.000
-12.000
.000
1.000
2.000
3.000
4.000
5.000
TIME SECONDS

FIG. 12B

ZERO SUPPRESSION

The Laplace-transformed analytical representation of the input data, equation 11, has one fewer zero than poles in each element of the matrix transfer function $\underline{G}(s)$ where

$$\underline{Y}(s) = \underline{G}(s)/s$$

or

$$Y_i(s) = \frac{c_{i1}}{s-\lambda_1} + \frac{c_{i2}}{s-\lambda_2} + \frac{c_{i3}}{s-\lambda_3} + \frac{c_{i4}}{s-\lambda_4} + \frac{co_i}{s} \quad .$$

This expression can be rewritten, omitting the i subscript

$$\begin{aligned}
&(c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4)s^3 - (c_3\lambda_3\lambda_4 + c_4\lambda_3\lambda_4 + c\lambda_1\lambda_3 \\
&+ c_3\lambda_1\lambda_3 + c_1\lambda_1\lambda_4 + c_4\lambda_1\lambda_4 + c_2\lambda_2\lambda_3 + c_2\lambda_2\lambda_4 + c_4\lambda_2\lambda_4 + c_1\lambda_2\lambda_1 \\
&+ c_2\lambda_1\lambda_2)s^2 - (c_1\lambda_1\lambda_3\lambda_4 + c_3\lambda_1\lambda_3\lambda_4 + c_4\lambda_1\lambda_3\lambda_4 + c_4\lambda_1\lambda_3\lambda_4 + c_2\lambda_2\lambda_3\lambda_4 \\
&+ c_3\lambda_2\lambda_3\lambda_4 + c_2\lambda_2\lambda_3\lambda_4 + c_1\lambda_1\lambda_2\lambda_3 + c_2\lambda_1\lambda_2\lambda_3 + c_3\lambda_1\lambda_2\lambda_3 \\
&+ c_1\lambda_1\lambda_2\lambda_4 + c_2\lambda_1\lambda_2\lambda_4 + c_4\lambda_1\lambda_2\lambda_4)s + \lambda_1\lambda_2\lambda_3\lambda_4
\end{aligned} \tag{31}$$

To reduce the number of zeros in the transfer function, the following constraint equations must be incorporated into the least-square minimization, equation 29. To suppress one zero

$$c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4 = 0 \quad . \tag{32}$$

To suppress two zeros

$$c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4 = 0$$

and

$$c_3\lambda_3\lambda_4 + c_4\lambda_3\lambda_4 + c_1\lambda_1\lambda_3 + c_3\lambda_1\lambda_3 + c_1\lambda_1\lambda_4 + c_4\lambda_1\lambda_4$$

$$+ c_2\lambda_2\lambda_3 + c_3\lambda_2\lambda_3 + c_2\lambda_2\lambda_4 + c_4\lambda_2\lambda_4 + c_1\lambda_1\lambda_2 + c_2\lambda_1\lambda_2 = 0 \quad (33)$$

If three zeros are suppressed, only one parameter remains to be established by the minimization and the resulting representation of the input time history is inadequate.

For low-order models, zero suppression has two detrimental effects on the models obtained. The first is the worsening of the agreement between the discretized input time histories and their analytical representations. For example, the unconstrained least-squared-error method yields a satisfactory fit of the roll-rate input, as shown in Figure 6. The continuous fitted curve has approximately the same relationship to the envelope as does the discretized input data. If the LSE method is constrained by equation 32 to have one less zero, one obtains the fit shown in Figure 13. If one further constrains the LSE minimization by equation 33, another zero will be eliminated and, for the example data, the fit is degraded to that shown in Figure 14. The same sequence is portrayed in Figures 7, 15, and 16 for the sideslip input. The Figure 7 results are unconstrained, one zero is suppressed in Figure 15 and two zeros are suppressed in Figure 16. Important features of the discrete inputs can be preserved by weighting the appropriate errors, e_z, in the LSE method. This may make the constrained fits more useful. Even the unconstrained cases can be altered. The risetime of the fitted roll rate can be reduced, for example. However, the overall fit cannot be improved by weighting.

The second detrimental effect of zero suppression is the increased tendency of the $\underline{A}$ matrix to contain unrealistic values. The $\underline{A}$ and $\underline{b}$ arrays resulting from unconstrained fits of the input time histories are

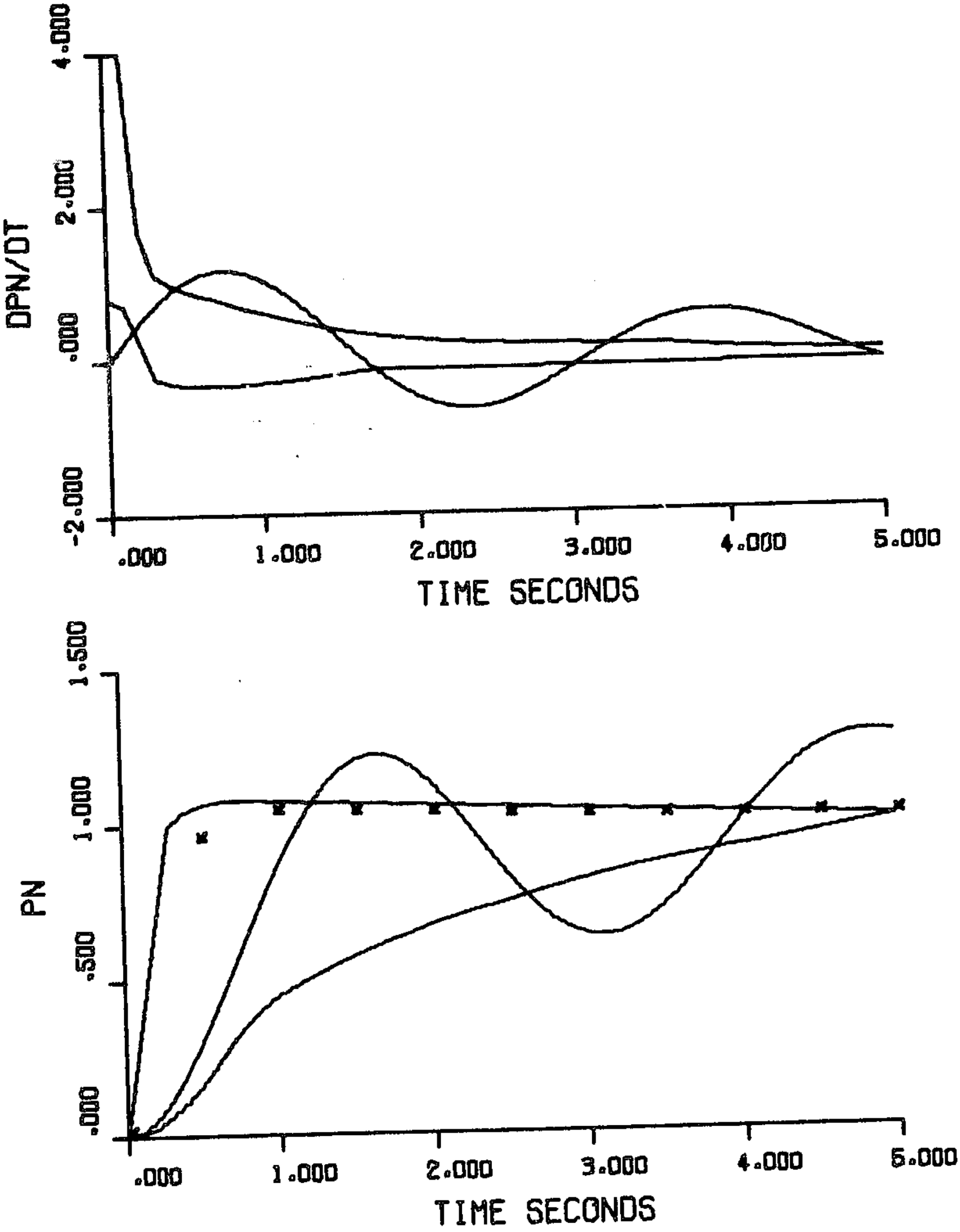


FIG.13 ROLLRATE

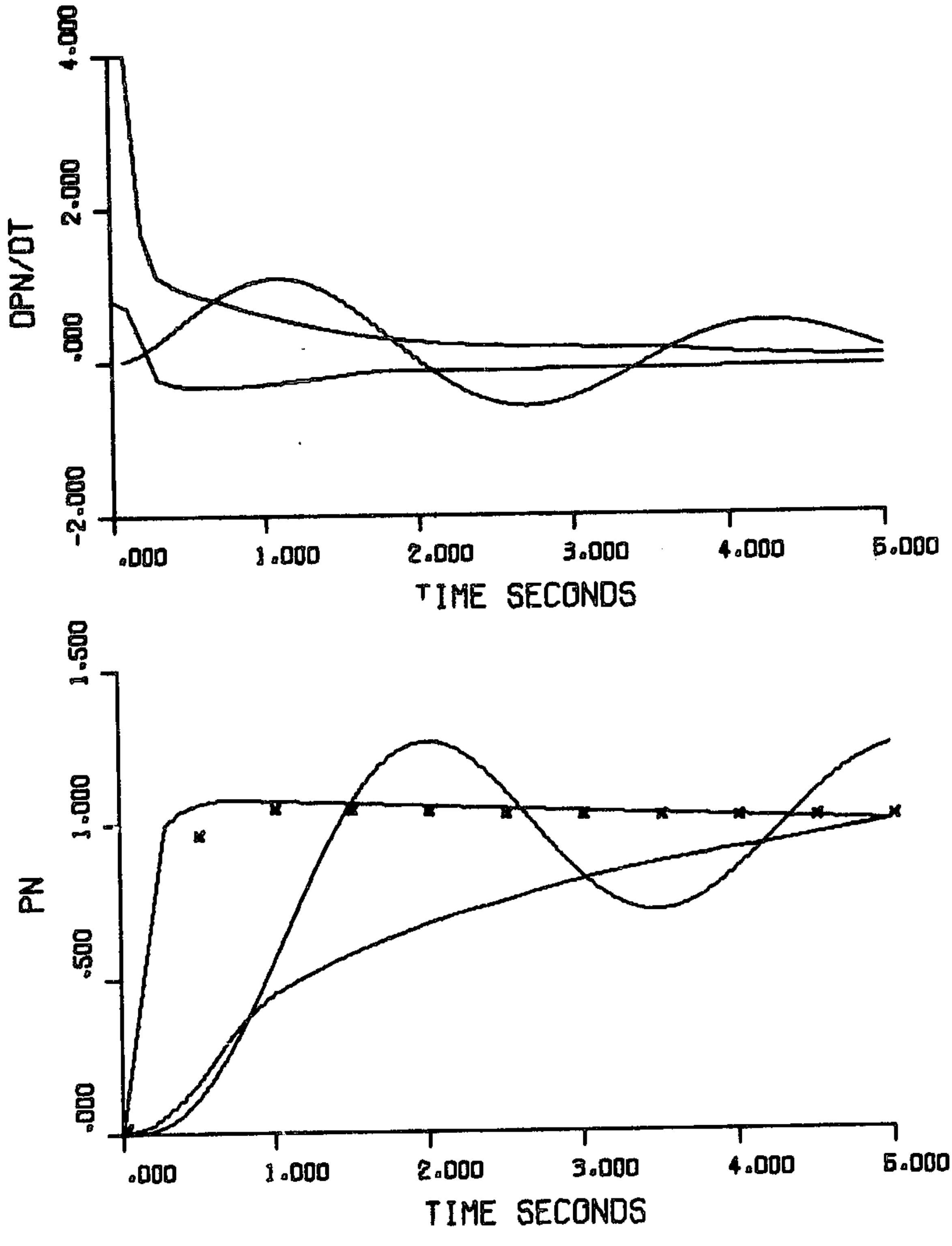


FIG.14 ROLLRATE

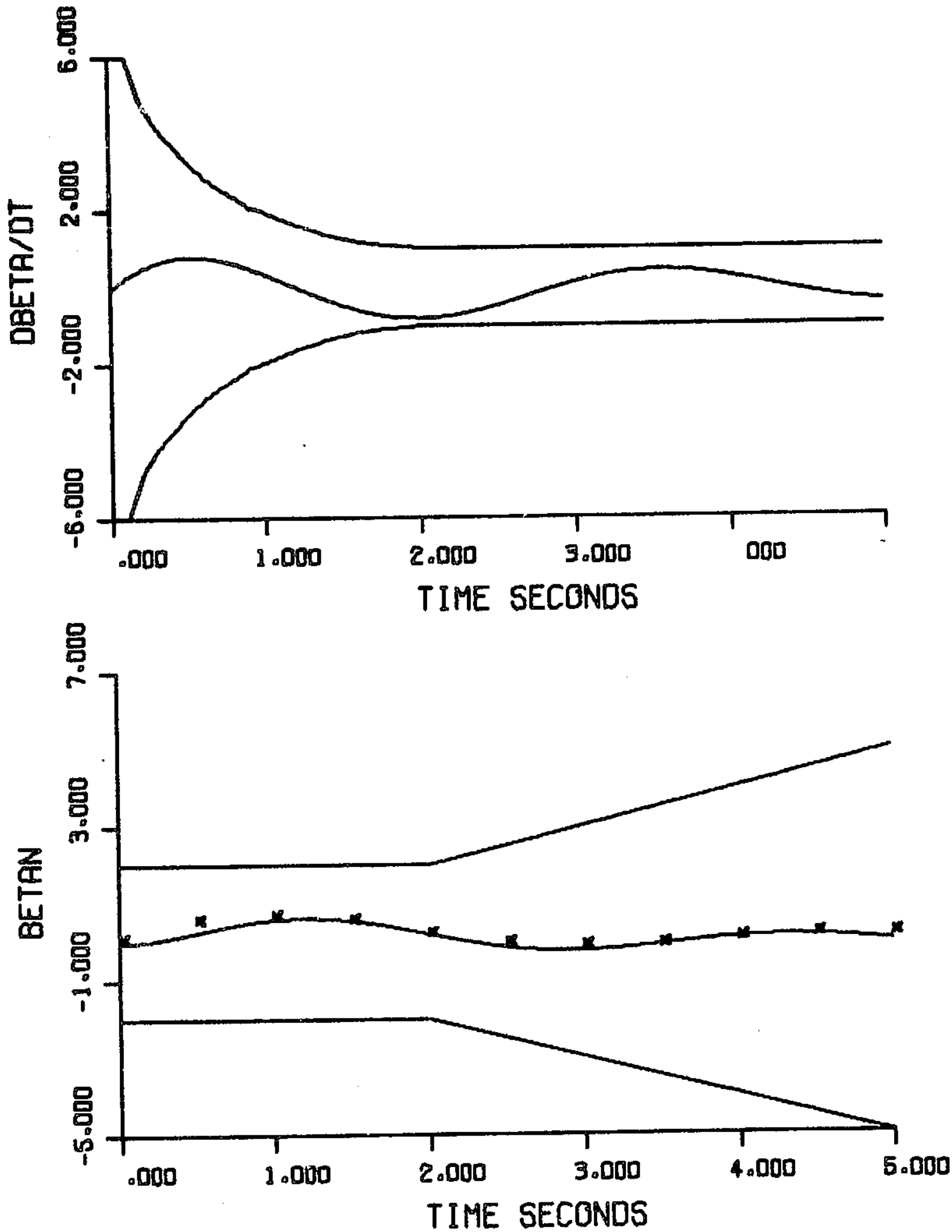


FIG.15 SIDESLIP

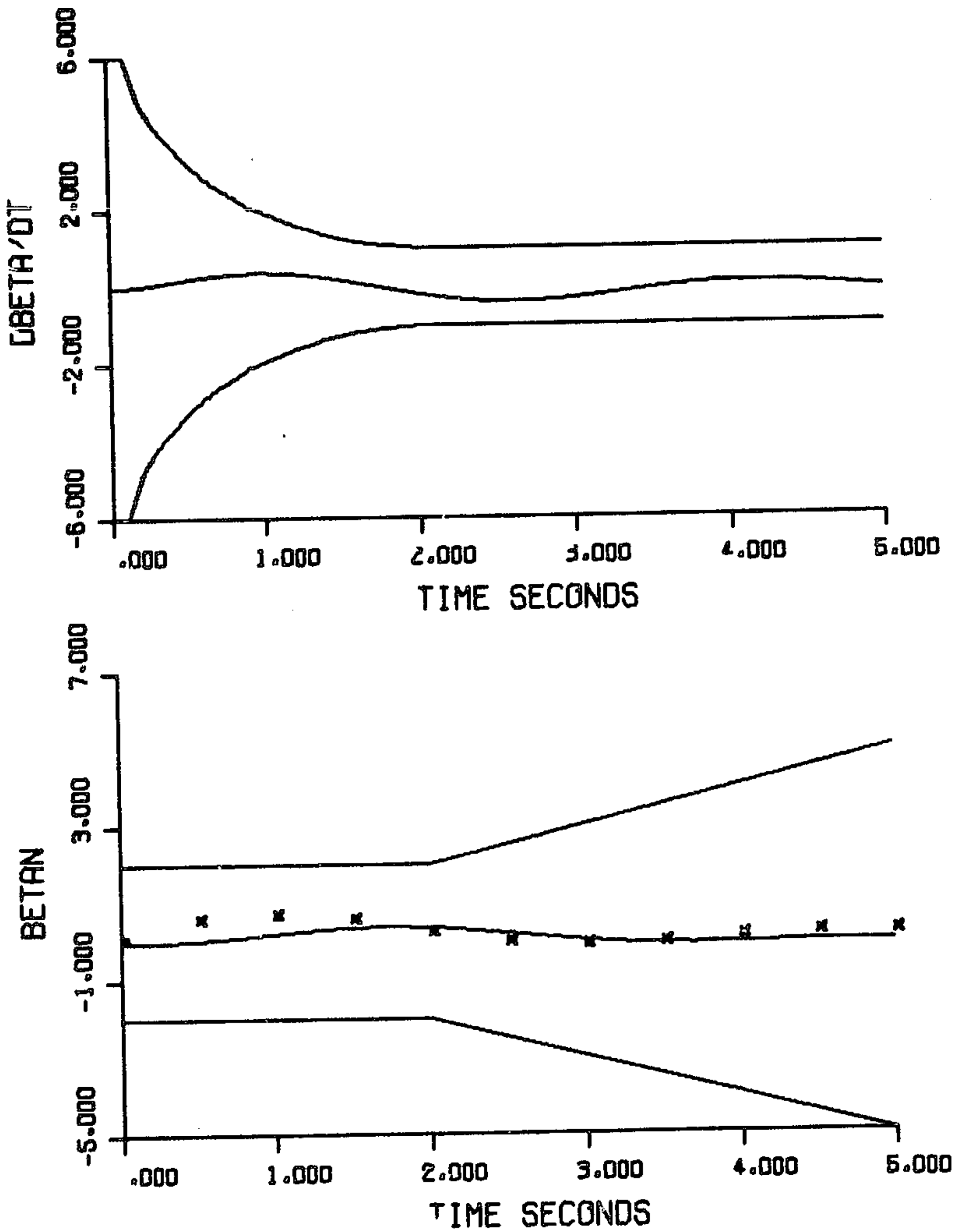


FIG.16 SIDESLIP

$$
\underset{\sim}{A} = \begin{bmatrix} -1.36 & -4.17 & -20.22 & -.02 \\ -1.12 & 1.89 & 12.43 & -.04 \\ 0.22 & -.80 & -3.43 & 0.02 \\ 0.20 & 1.04 & 5.61 & 0.00 \end{bmatrix}
$$

and

$$
\underset{\sim}{b} = \begin{bmatrix} 5.06 \\ 1.59 \\ -.12 \\ 1.09 \end{bmatrix}
$$

The Newton-Euler procedure for obtaining the a_{ij} converged in three iterations to a maximum variation of a[illegible]ment from one iteration to the next of .00001 or less. This model was obtained from the unconstrained fits shown in Figures 6, 7, and 8.

If one zero in the roll-rate transfer function and one zero in the side-slip transfer function are suppressed, the $\underset{\sim}{A}$ and $\underset{\sim}{b}$ arrays are altered to

$$
\underset{\sim}{A} = \begin{bmatrix} -3.49 & 4.73 & 97.14 & 0.52 \\ 0.96 & -3.42 & -22.48 & -.07 \\ -.19 & 0.30 & 4.00 & 0.02 \\ -.32 & 2.44 & 15.42 & 0.01 \end{bmatrix}
$$

and

$$
\underset{\sim}{b} = \begin{bmatrix} 0.00 \\ 1.06 \\ 0.00 \\ 0.87 \end{bmatrix}
$$

The Newton-Euler procedure again required three iterations. This model was

obtained from the fits shown in Figures 13, 15, and 8.

If two zeros in the roll-rate transfer function and two zeros in the sideslip transfer function are suppressed, the $\underset{\sim}{A}$ and $\underset{\sim}{b}$ arrays are

$$\underset{\sim}{A} = \begin{bmatrix} -9.34 & -1.66 & 403.88 & 2.75 \\ 3.51 & -2.88 & 138.62 & -.86 \\ -.21 & -.04 & 8.73 & 0.06 \\ -2.10 & 1.45 & 96.10 & 0.59 \end{bmatrix}$$

and

$$\underset{\sim}{b} = \begin{bmatrix} 0.00 \\ 1.45 \\ 0.00 \\ 0.87 \end{bmatrix}$$

Three iterations were sufficient and the model was obtained from the data shown in Figures 14, 16, and 8.

The number of terms in the expansion of the state transition matrix specified by Paynter's recipe, equation 30, is a function of the largest absolute value contained in $\underset{\sim}{A}$ and this number is frequently larger than the practical limit of about 30 when zeros are suppressed.

The additional information required to calculate $\underset{\sim}{A}$ and $\underset{\sim}{b}$ from the discrete time histories shown in Figures 7-16 is given in Table 2. The specified eigenvalues were

$$\lambda_1 = -2.4$$

$$\lambda_2 = -.003$$

$$\lambda_{3,4} = -.25 \pm j2.0$$

AIRPLANE MODEL WITH SPECIFIED TIME HISTORIES

FLIGHT AND VEHICLE PARAMETERS

AIRSPEED 612.2 FT/SEC

CG TO PILCT STATICN LONGITUDINAL DISTANCE 22.24 FT

DIMENSIONAL CONSTANT FOR DSTAR EQUATION -.3190 CUBIC-FEET/LB-SECONDS-SQUARED

DYNAMIC PRESSURE 331.8 LB/FT-SQUARED

ROLLRATE NORMALIZATION FACTOR .500

SIDESLIP NORMALIZATION FACTOR 10.000

DSTAR NORMALIZATION FACTOR .010

Jetstar Parameters and Flight Conditions

Table 2

The eigenvalues and vehicle and flight parameters are approximately those of a Lockheed Jetstar, a four-engined utility transport [7], at 20,000 feet altitude and Mach = 0.6. It should be noted that the discrete input time histories were obtained from p_n, β_n, and D_n^* responses sketched by an FRC engineer [8] and do not refer to a particular airplane or flight condition. Finally, FRC program CONTROL was used to calculate transfer function coefficients from the above $\underline{A}$ and $\underline{b}$ arrays in combination with the appropriate distribution matrices, calculated from equation 22, to verify the suppression of the specified zeros.

VERIFICATION

Flight test data obtained from the FRC Lockheed Jetstar [9] provides a test case for the above model-generation procedure. For small, lateral variations about straight and level flight, the Jetstar can be represented by*

$$\underline{A} = \begin{bmatrix} -2.353 & 0.735 & -11.050 & 0.000 \\ -.057 & 0.358 & 3.836 & 0.000 \\ 0.026 & -.999 & -.205 & 0.053 \\ 1.000 & 0.054 & 0.000 & 0.000 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 5.650 \\ 0.031 \\ -.001 \\ 0.000 \end{bmatrix}$$

*Body axis nondimensional stability derivative parameters used in place of stability axis values for verification purposes only.

$$\underset{\sim}{\tilde{G}} = \begin{bmatrix} 1. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \\ 14.75 & -7.044 & -146.0 & 32.14 \end{bmatrix}$$

$$\underset{\sim}{\tilde{h}} = \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \end{bmatrix}$$

The eigenvalues were

$$\lambda_1 = -.24045$$

$$\lambda_2 = -.00310$$

$$\lambda_{3,4} = -.25428 \pm j2.06475$$

CONTROL was used to calculate the handling-quality time histories for the period 0.0 to 5.0 seconds. The discrete input data extracted from the CONTROL output is given in Table 3. The flight and vehicle parameters are given in Table 2. The normalization factors were arbitrarily selected.

The responses of the resulting model are shown in Figures 17, 18, and 20 and compared with the discrete input values of Table 3 after normalization. The $\underset{\sim}{A}$, $\underset{\sim}{b}$, $\underset{\sim}{\tilde{G}}$ and $\underset{\sim}{\tilde{h}}$ arrays which define the model are

$$\underset{\sim}{A} = \begin{bmatrix} -2.359 & 0.771 & -10.939 & -.003 \\ -.053 & -.359 & 3.849 & 0.000 \\ 0.025 & -1.005 & -.201 & 0.053 \\ 0.906 & -.021 & .278 & 0.003 \end{bmatrix}$$

Table 3

Time	Rollrate	Sideslip	D^*
0.0	0.00	0.00	0.0
0.5	1.64	.016	37.6
1.0	2.04	.058	67.6
1.5	2.04	.093	95.1
2.0	2.01	.098	126.
2.5	2.06	.080	161.
3.0	2.14	.065	198.
3.5	2.18	.069	234.
4.0	2.15	.089	265.
4.5	2.09	.109	295.
5.0	2.05	.115	327.

These values are multiplied by the normalization factors listed in Table 2 before they are plotted.

Jetstar Discrete Time-History Data

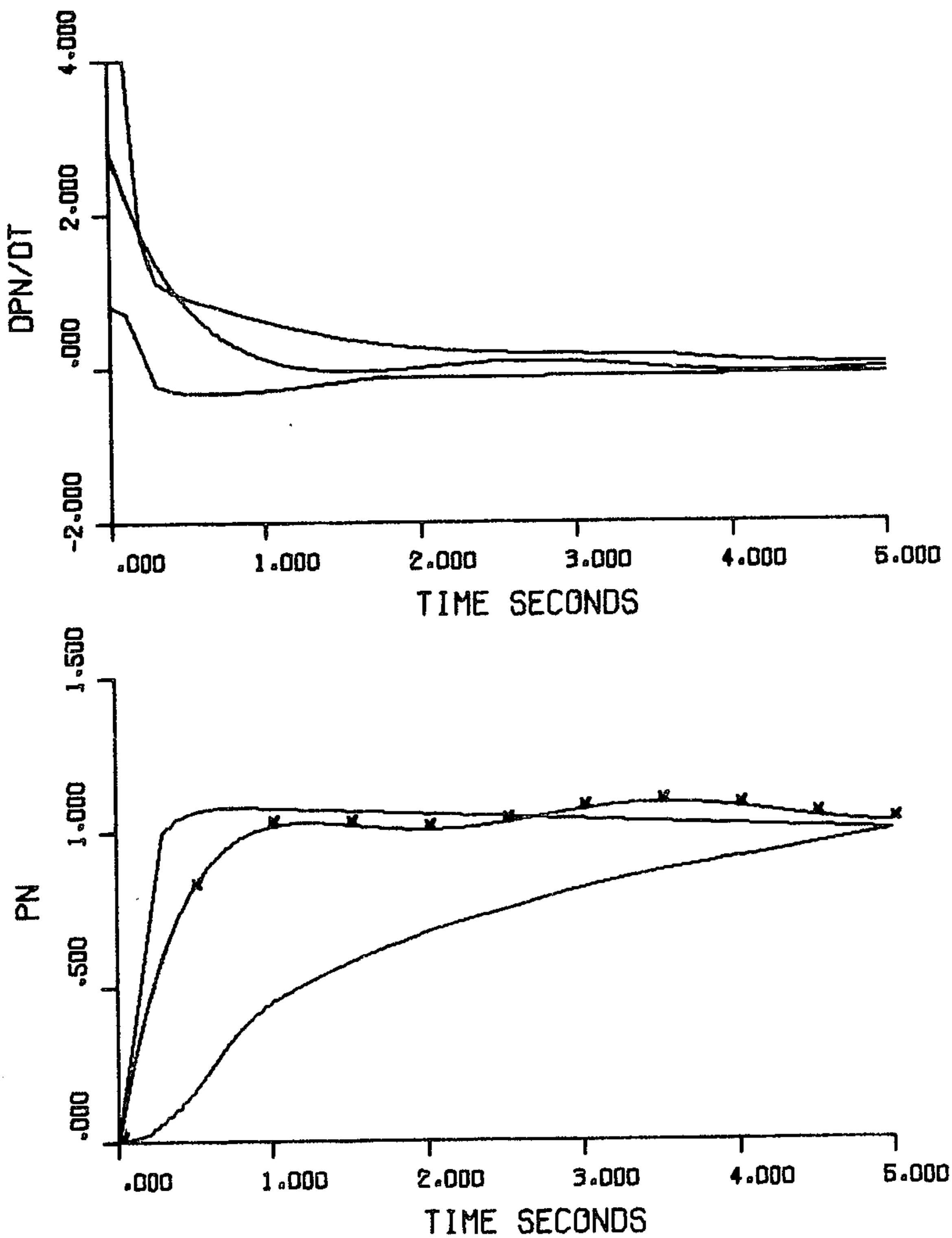


FIG. 17 ROLLRATE

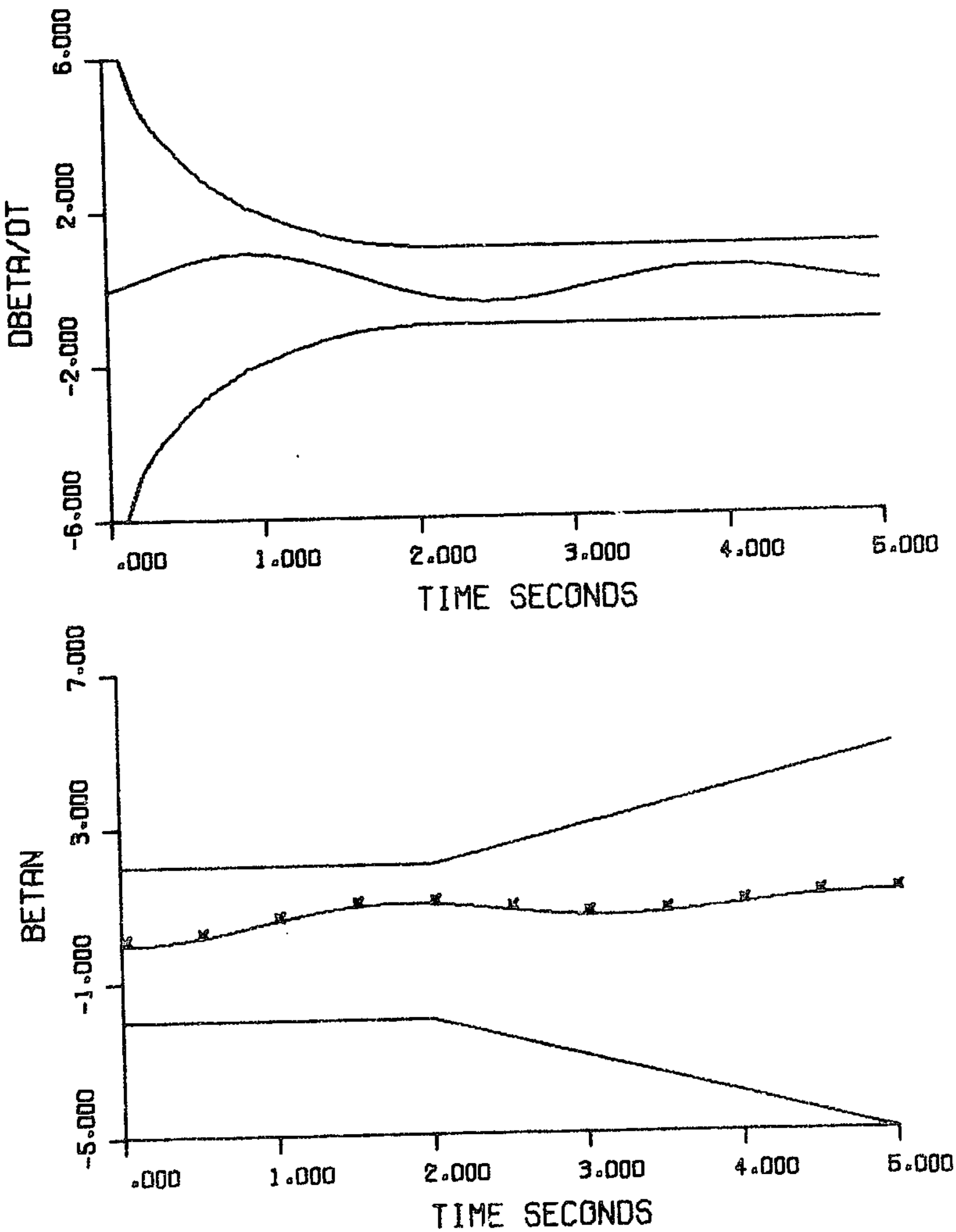


FIG.18 SIDESLIP

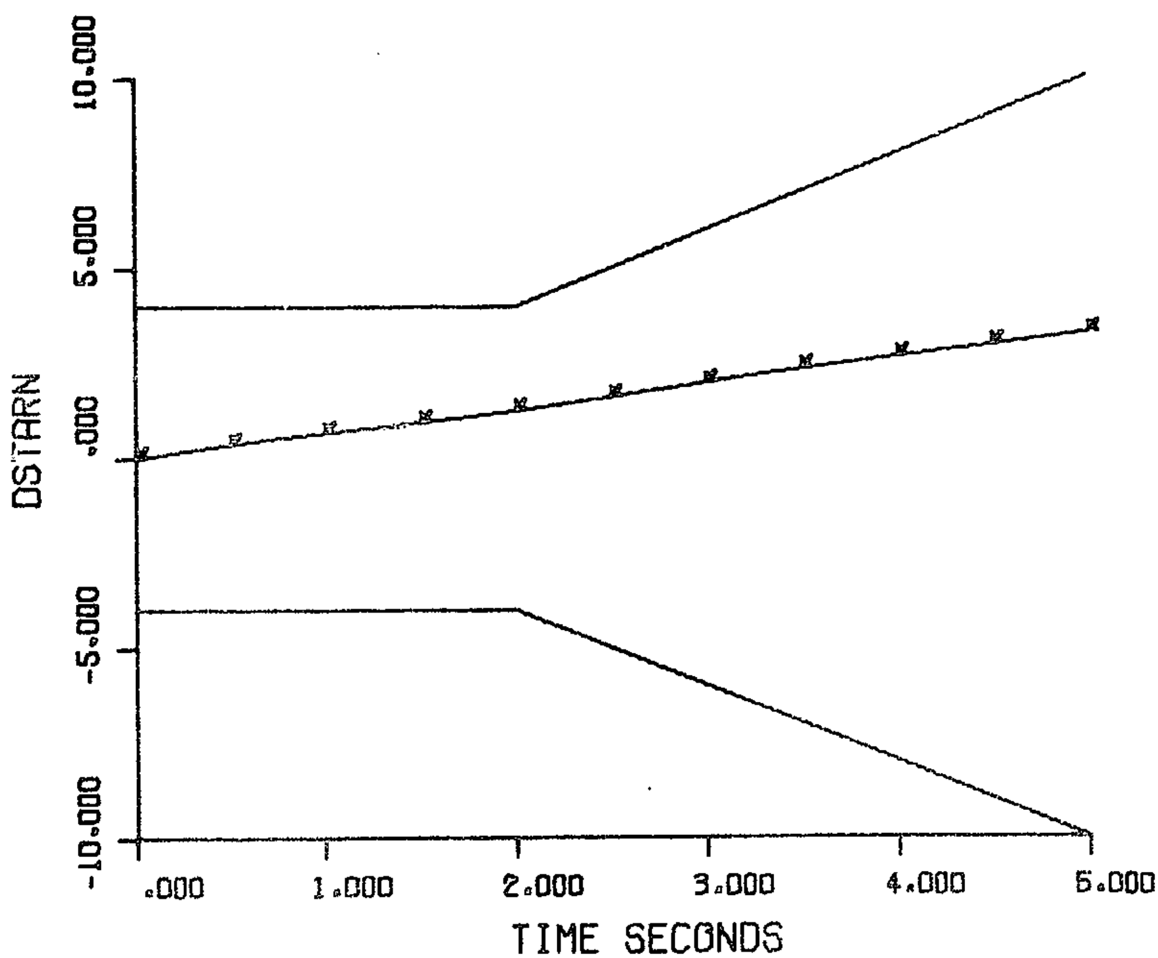
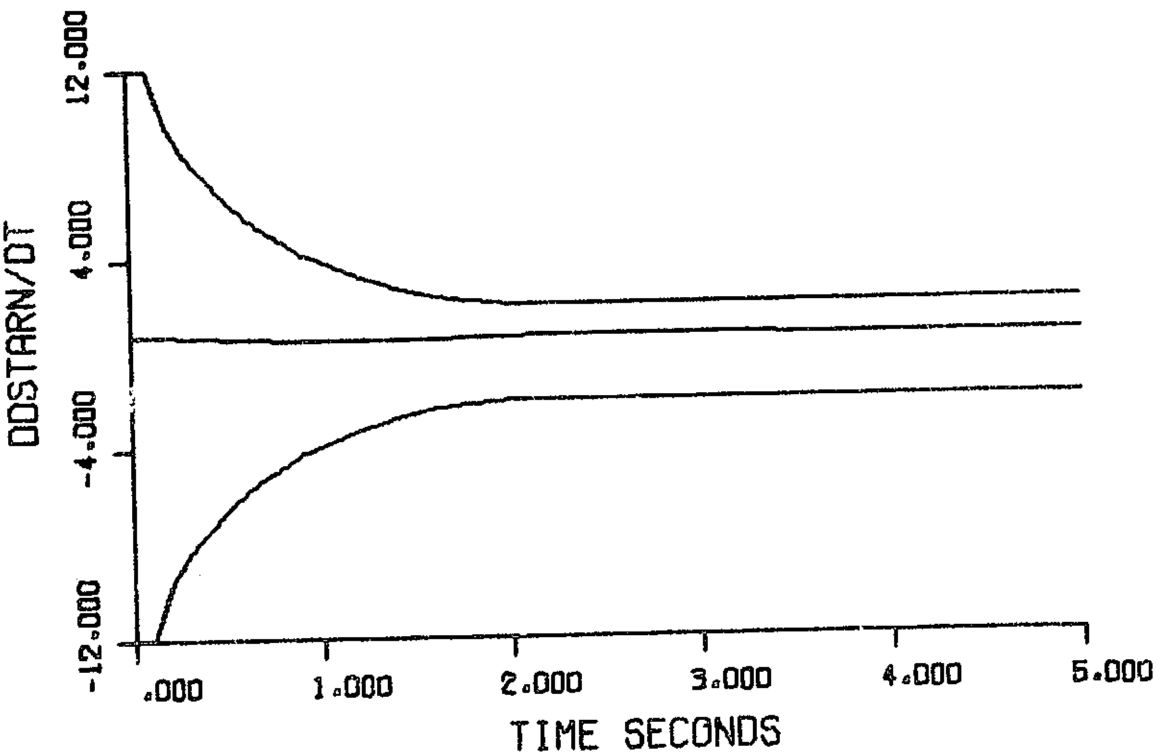


FIG.19A LATERAL CRITERION



DDSTARN/DT
12.000
4.000
-4.000
-12.000
.000
1.000
2.000
3.000
4.000
5.000
TIME SECONDS

FIG. 19B

$$\underset{\sim}{b} = \begin{bmatrix} 5.818 \\ -.053 \\ -.215 \\ 0.050 \end{bmatrix}$$

$$\underset{\sim}{\tilde{G}} = \begin{bmatrix} 1.0 & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \\ 14.35 & -11.13 & -143.5 & 32.61 \end{bmatrix}$$

$$\underset{\sim}{\tilde{h}} = \begin{bmatrix} 0. \\ 0. \\ 0. \\ -.023 \end{bmatrix}$$

These results are close to the original arrays. The differences are attributed to the truncation of the CONTROL output values to two or three significant figures to approximate imprecise or sketched input data.

DISCUSSION

The problem of obtaining an $\underline{A}$ and $\underline{b}$ from specified output time histories is one of nonlinear, noisefree identification. Five techniques for solving this problem have been suggested. The minimization of a cost functional which measures the differences between a trial solution and the handling-quality time-history envelopes would consume a large amount of computer time and there is no assurance that such a cost functional would be sufficiently well behaved to have a useful solution. Similarly, one could consume a large amount of computer time seeking solutions by random direct search. A graduate student is currently working on a variation of direct search in which the sensitivity of the time-history errors to changes in the $\underline{A}$ matrix elements is calculated. Then a set of incremental changes to the $\underline{A}$ elements can be obtained.

The Laplace transformation method is also being pursued by a graduate student. In this form the problem is quite easily formulated but is ill-posed. It remains to be seen whether this method can advantageously incorporate the loose bounds on the eigenvalues or not. It also has the disadvantage of being a two-approximate-step process. One obtains $\underline{y}(t)$ from $\hat{\underline{y}}(t)$ and then $\tilde{\underline{y}}(t)$ from $\underline{y}(t)$ except that the latter two will not coincide as they do in the pseudodata method. The Laplace transformation method and the sensitivity matrix method do have the advantage that they can be made to yield $\underline{A}$ matrices of the form of equation 19.

The remaining approach, the pseudodata method has two advantages. It contains only one approximation step and it is numerically efficient. The disadvantages are that it is somewhat less general and yields unconstrained $\underline{A}$ matrices. Experience has shown that the transfer functions resulting from

the unconstrained $\underline{A}$ matrices resemble those produced by $\underline{A}$ matrices of the form of equation 19. The pseudodata method results in a well-posed set of bilinear algebraic equations which yield an $\underline{A}$ matrix having the specified eigenvalues.

The pseudodata method is the only method which readily achieves zero suppression. In the other methods zero suppression contributes to their ill-posedness making useful solutions even more numerically difficult to obtain. The utility of any zero-suppressed solution is called into doubt by the detrimental effect suppression has on the LSE fit of the discretized input data (see Figures 13, 14, 15, and 16). Relieved of the need to suppress transfer-function zeros, one might prefer one of the other methods.

APPENDIX A

Use of Program AANDB

INTENDED OPERATION

1. Put in normalized discrete data
2. Obtain normalized responses
3. Put in normalization factors
4. Obtain the normalizing distribution matrix
5. Obtain the non-normalized airplane equations in state-space form.
6. Use CONTROL to obtain transfer functions

Steps 1 through 5 are performed by an example Fortran program AANDB for the fourth-order small-lateral-motion case. The structure of the program is shown in Figure A-1. The subroutines perform the following tasks:

MUGEN: Calculates eigenvalues from input time-history data.

CASE1* (with entry points CASE2 and CASE3): Calculates the c_i required to fit $c_1e^{\lambda_1 t} + c_2e^{\lambda_2 t} + c_3e^{\lambda_3 t} + c_4e^{\lambda_4 t} + c_5$ to the input time-history data.

LISTER: Prints information from COMMON on demand.

FITTING: Fits a polynomial to roll-rate data and integrates it to produce roll-angle pseudodata.

MODEL: Calculates the plant matrix, $\underline{A}$, input distribution vector, $\underline{b}$, the output distribution matrix, $\underline{G}$, and the input/output coupling vector, $\underline{h}$.

RESPONS: Integrates the plant equations to produce comparison time histories.

---

*CASE1 does not suppress any transfer function zeros, CASE2 suppresses one zero and CASE 3 suppresses two zeros.

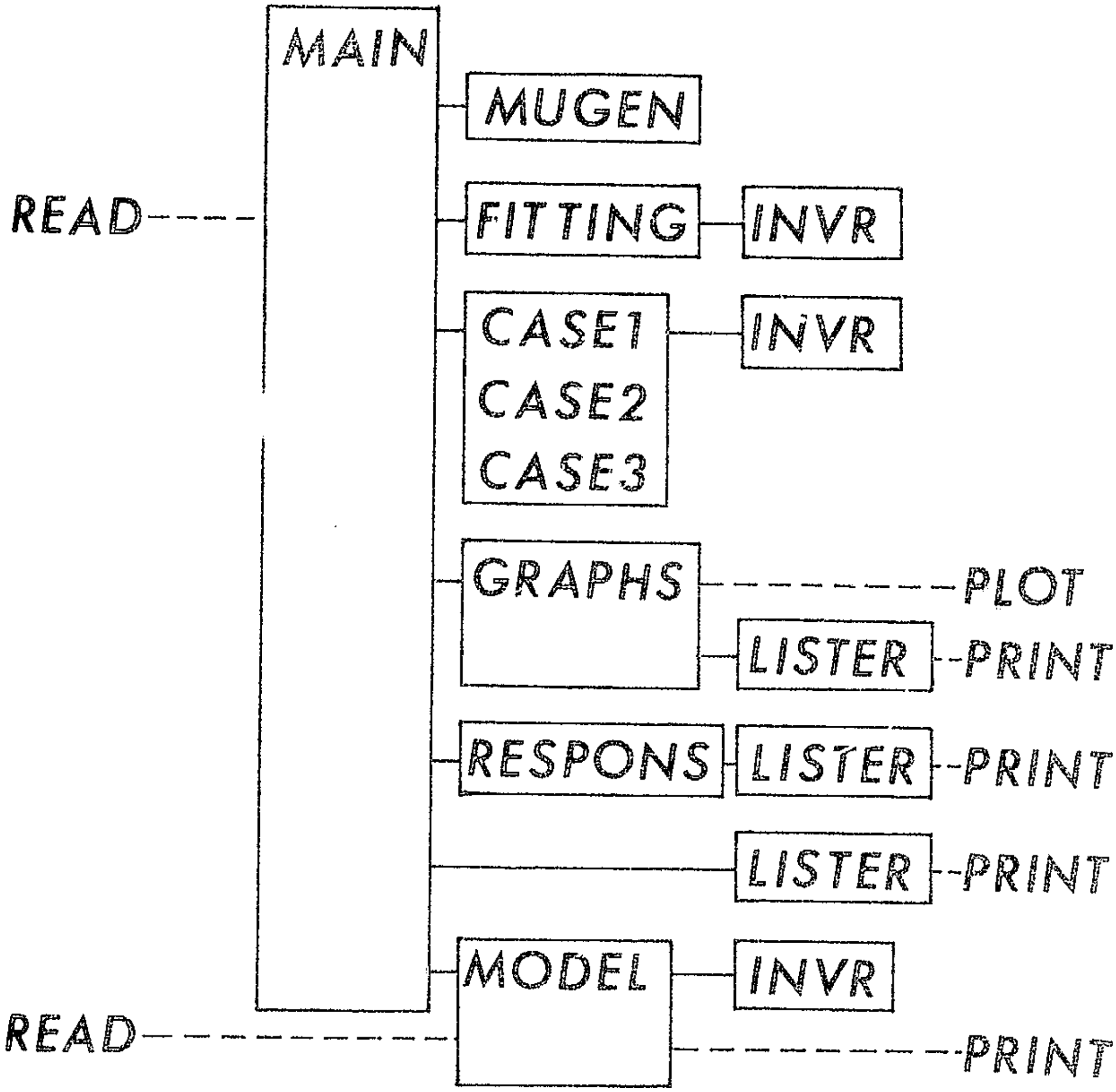

MAIN
READAD
MUGEN
FITTING
INVR
CASE1
CASE2
CASE3
INVR
GRAPHS
PLOT
LISTER
PRINT
RESPONS
LISTER
PRINT
LISTER
PRINT
MODEL
INVR
READ
PRINT

FIG. A1

GRAPHS: Plots envelopes and time histories and their first derivatives.

INVR: Inverts matrices.

The first fourteen input data cards are read by MAIN. The final two input data cards are read by subroutine MODEL. The input data cards contain the following information:

<u>1st data card</u>: NN(20); 20I2

IF NN(1) = 1: LISTER will print: data, NN, MCASE, NPLOT, NHIST, input time histories

NN(2) = 1: LISTER will print eigenvalues obtained from input time histories

NN(3) = 1: LISTER will print eigenvalues specified by user

NN(4) = 1: LISTER will print roll angle data obtained from roll rate

NN(5) = 1: LISTER will print complex coefficient matrix $\underline{C}$

NN(6) = 1: LISTER will print summary of time-history curve fitting results

NN(7) = 1: not used

NN(8) = 1: not used

NN(9) = 1: LISTER will print envelopes for PN, BETAN, DSTAR & derivatives

NN(10) = 1: LISTER will print fitted curves for PN, BETAN, DSTAR & derivatives

NN(11) = 1: LISTER will print number of terms included in series for $e^{\underline{A}\Delta t}$

NN(12) = 1: LISTER will print difference equation $\underline{P}$ matrix

NN(13) = 1: LISTER will print difference equation $\underline{q}$ vector

NN(14) = 1: LISTER will print responses obtained by integrating model equations

NN(15) = 1: LISTER will print derivatives of responses obtained from $\underline{P}$, $\underline{q}$

NN(16) = 1: LISTER will print "PN PLOTTED" if NPLOT requests it

NN(17) = 1: LISTER will print "BETAN PLOTTED" if NPLOT requests it

NN(18) = 1: LISTER will print "DSTAR PLOTTED" if NPLOT requests it

NN(19) = 1: not used

NN(20) = 1: not used

2nd - 12th data cards: TIME, PN, BETAN, DSTAR; 4F10.0

0.0	0.0	0.0	0.0
0.5	PN(.5)	BETAN(.5)	DSTAR(.5)
1.0	PN(1.)	BETAN(1.)	DSTAR(1.)
1.5	PN(1.5)	BETAN(1.5)	DSTAR(1.5)
2.0	PN(2.)	BETAN(2.)	DSTAR(2.)
2.5	PN(2.5)	BETAN(2.5)	DSTAR(2.5)
3.0	PN(3.)	BETAN(3.)	DSTAR(3.)
3.5	PN(3.5)	BETAN(3.5)	DSTAR(3.5)
4.0	PN(4.)	BETAN(4.)	DSTAR(4.)
4.5	PN(4.5)	BETAN(4.5)	DSTAR(4.5)
5.0	PN(5.)	BETAN(5.)	DSTAR(5.)

13th data card: MCASE, NPLOT, NHIST; 3I4

If NHIST ≠ 0 subroutine RESPONS will be called to calculate the model time histories. If NPLOT ≠ 0 subroutine graphs will be called to plot the model time histories according to:

NPLOT

0 no plots
1 PN and PNDOT plotted
2 BETAN and BETANDOT plotted
3 PN, PNDOT, BETAN, BETANDOT plotted
4 PSTAR, DSTARDOT plotted
5 BETAN, BETANDOT, DSTAR, DSTARDOT plotted
6 PN, PNDOT, DSTAR, DSTARDOT plotted
7 all plots
8 no plots

MCASE must be appropriate for:

```
GO TO(1,2,3,4,5,6,7,8,9), MCASE
```

1 P(s) has no zero(s) suppressed
  β(s) has no zero(s) suppressed

2 P(s) has two zero(s) suppressed
  β(s) has one zero(s) suppressed

3 P(s) has one zero(s) suppressed
  β(s) has two zero(s) suppressed

4 P(s) has one zero(s) suppressed
  β(s) has one zero(s) suppressed

5 P(s) has one zero(s) suppressed
  β(s) has no zero(s) suppressed

6 P(s) has no zero(s) suppressed
  β(s) has one zero(s) suppressed

7 P(s) has two zero(s) suppressed
  β(s) has no zero(s) suppressed

8 P(s) has no zero(s) suppressed
  β(s) has two zero(s) suppressed

9 P(s) has two zero(s) suppressed
  β(s) has two zero(s) suppressed

14th data card: specified eigenvalues; 8F10.0

1-10 First eigenvalues which is real
21-30 Second eigenvalue which is also real
41-50 Real part of third eigenvalue
51-60 Imaginary part of third eigenvalue
61-70 Real part of fourth eigenvalue
71-80 Imaginary part of fourth eigenvalue

15th card: velocity, length, c_3, q_{co}, P_{ss}, β_{ss}, D^*_{ss}; 7F10.0

VELOCITY: Nominal airspeed in ft/sec

LENGTH: Centerline length from CG to pilot in ft

c_3: Dimensional constant for D^*

q_{co}: Nominal dynamic pressure in lb/ft^2

p_{ss}: Roll rate normalization factor

β_{ss}: Sideslip normalization factor

D^*_{ss}: DSTAR normalization factor

16th data card: Newton-Euler parameters; I4, F20.0

ITMAX, I4, Maximum number of iterations of Newton-Euler algorithm

EPSI, F20.0, when every unknown (the elements of the A matrix) changes by an amount smaller than EPSI, the Newton-Euler Algorithm stops

Note: 50,0.00001 seem to work well.

INTERPRETATION OF THE PRINTED OUTPUT

If NN = 20*1 all of the following output will be produced:

LISTER calls from the main program print:

1. Date, list table NN, case number, plot request code, time response code and the input time history data.

2. Real and imaginary parts of each MU(4) and EI(4) obtained from the input time histories. The EI are eigenvalues, λ_i, and each MU is $\mu_i = e^{0.5\lambda_i}$ where 0.5 is the uniform time interval between input time-history data points.

3. Real and imaginary parts of the specified eigenvalues and associated MU values. These are the eigenvalues used in the least-square fitting process to obtain $\underline{C}$, not the quantities derived from the data. The derived values are presented only for comparison purposes.

4. The PHI or roll-rate psudodata generated from the PN or roll rate input time history. The time intervals are the same as for the original data.

5. The TIME RESPONSE COEFFICIENTS matrix, $\underline{C}'$, is printed. This is a 4 x 5 array of complex numbers:

$$\underline{y} = \underline{C}\underline{e}^{\underline{\Lambda}t} + \underline{c} \quad , \quad \text{where } \underline{C}' = [\underline{C} \vdots \underline{c}] \quad ,$$

which analytically represents the input time histories. This $\underline{y}(t)$ is printed as: FITTED TIME RESPONSES by a call to LISTER from RESPONS.

6. A summary printout of MU, EI, $\underline{C}$ and $\underline{c}$

LISTER calls from subroutine RESPONS print:

1. The envelopes, upper and lower boundaries, for p_n β_n, D^*, $\frac{d}{dt} p_n$, $\frac{d}{dt} \beta_n$, and $\frac{d}{dt} D^*$. Values are given for every 0.1 seconds. These values are stored

in DATA statements in the main program.

2. The curves which have been fitted to the input time-history data are printed for p_n, β_n and D^*. The analytic expressions for the curves are differentiated and tabulated also giving $\frac{d}{dt} p_n$, $\frac{d}{dt} \beta_n$ and $\frac{d}{dt} D^*$.

3. The number of terms taken to calculate the truncated series used to represent $e^{\underline{A}\Delta t}$ is PAYNTERS RECIPE NUMBER. See CONTROL, Takahashi, et. al., page 103 [6].

4. The difference equation parameters $\underline{P}$ and $\underline{q}$ in:

$$\underline{x}_{k+1} = \underline{\underline{P}}\underline{x}_k + \underline{q}u_k \quad , \quad \underline{x}_o = \begin{bmatrix} 0 \\ D_{\delta_a}/D_r \\ 0 \\ 0 \end{bmatrix}$$

5. The numerically integrated response time histories $\underline{y}_k$ where $\underline{y}_k = \underline{\underline{G}}\underline{x}_k + \underline{h}u_k$.

6. The first derivatives of the integrated responses are tabulated at 0.1 second intervals. They are obtained from

$$\dot{\underline{x}}_k = \underline{\underline{A}}\underline{x}_k + \underline{b}u_k$$

and

$$\dot{\underline{y}}_k = \underline{\underline{G}}\dot{\underline{x}}_k$$

LISTER calls from subroutine GRAPHS prints:

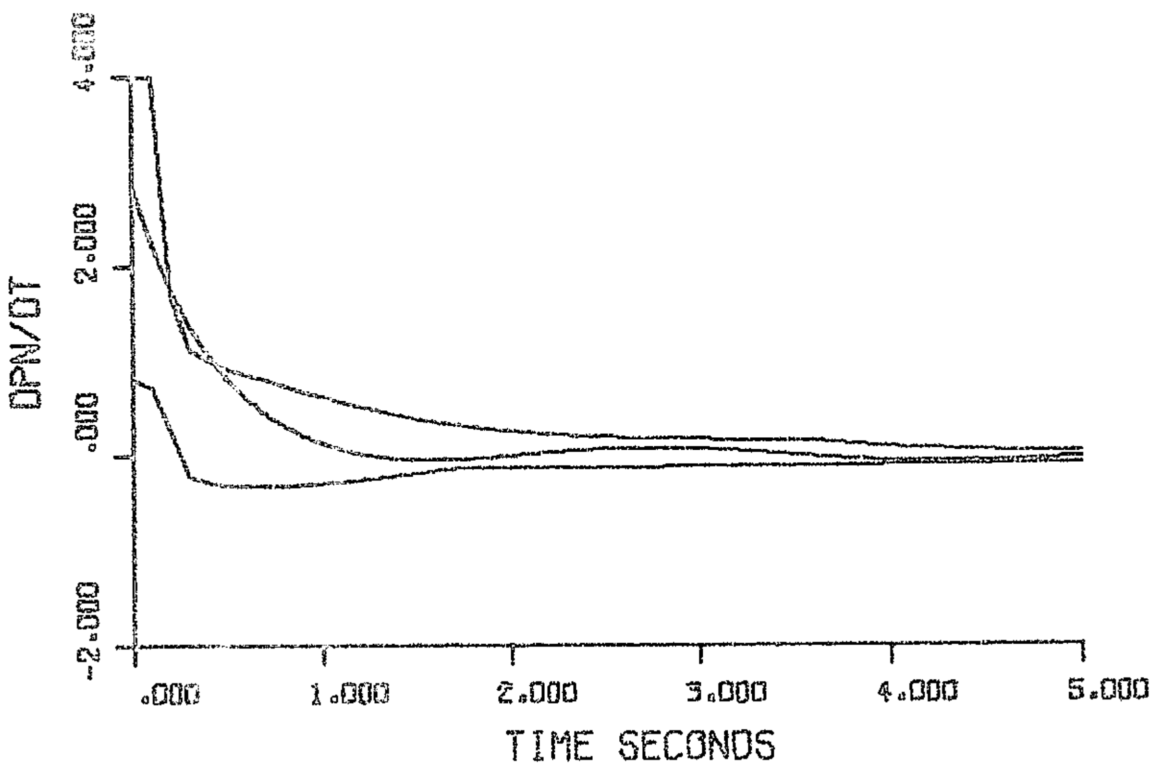
1. If NPLOT is such that plots are requested, LISTER will print "PN

PLOTTED" if ρ_n and $\frac{d}{dt}\rho_n$ plots have been generated, "BETAN PLOTTED" if β_n and $\frac{d}{dt}\beta_n$ plots have been generated and "DSTAR PLOTTED" if D^* and $\frac{d}{dt}D^*$ plots have been generated. The envelopes are automatically added to each plot.

SAMPLE PLOTTED OUTPUT

Plots are produced in three groups which can be requested individually or in any combination. These groups, the PN group, the BETAN group and the DSTAR group, consist of a title line, a lower graph and an upper graph. Each lower graph shows the upper envelope boundary, the lower envelope boundary, the analytical curve which has been fitted to the input data and the time history obtained by integrating the equations of motion. These four traces are represented by continuous lines and are plotted versus time on the horizontal scale. The fitted and integrated lines should coincide. In addition, the lower graph contains eleven discrete symbols representing the input data.

Each upper graph shows the upper envelope boundary, the lower envelope boundary, the first derivative of the fitted analytical curve and the first derivative of the integrated time response. The last two should coincide. All four curves are represented by continuous lines and are plotted versus time on the horizontal scale. A PN group sample is shown in Figure A-2, a BETAN group sample is shown in Figure A-3 and a DSTAR group is shown in Figure A-4. The example plots are unusual in that the original data was generated by a linear simulation program using a fourth-order model. Thus one should expect excellent agreement between the input data and the fitted and integrated results. Data obtained from other sources will not be matched as well.



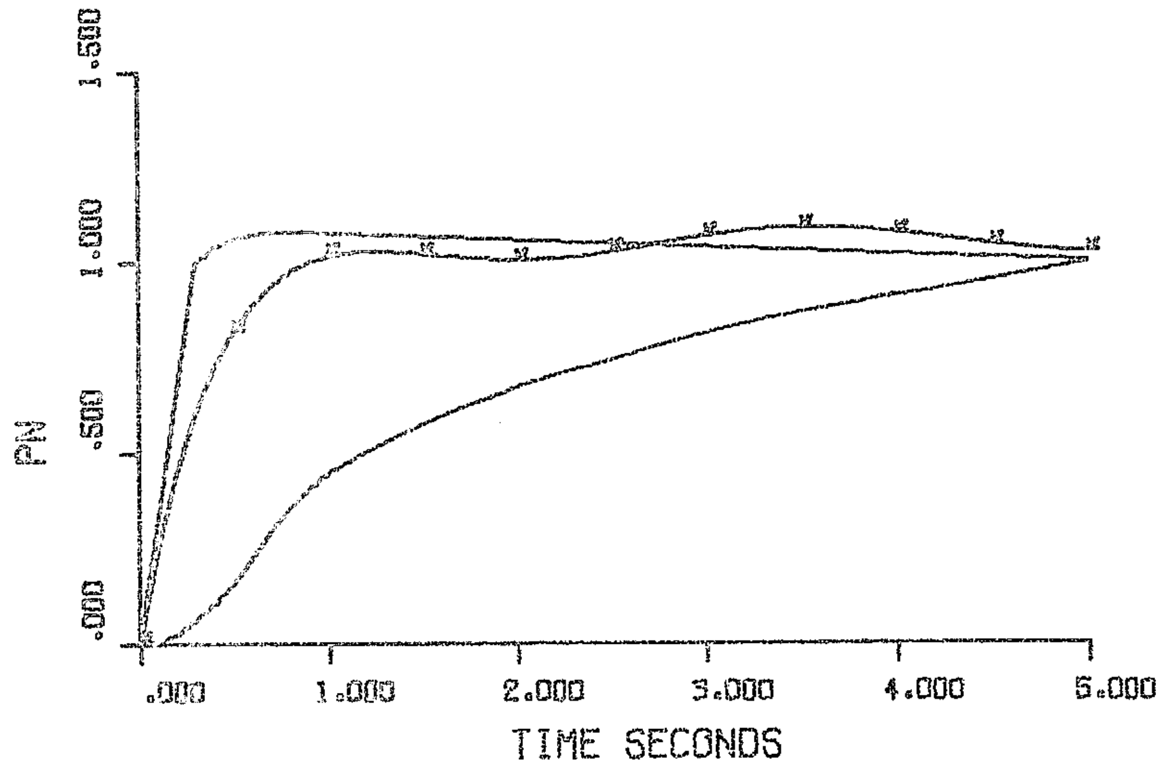


FIG. A2 ROLL RATE

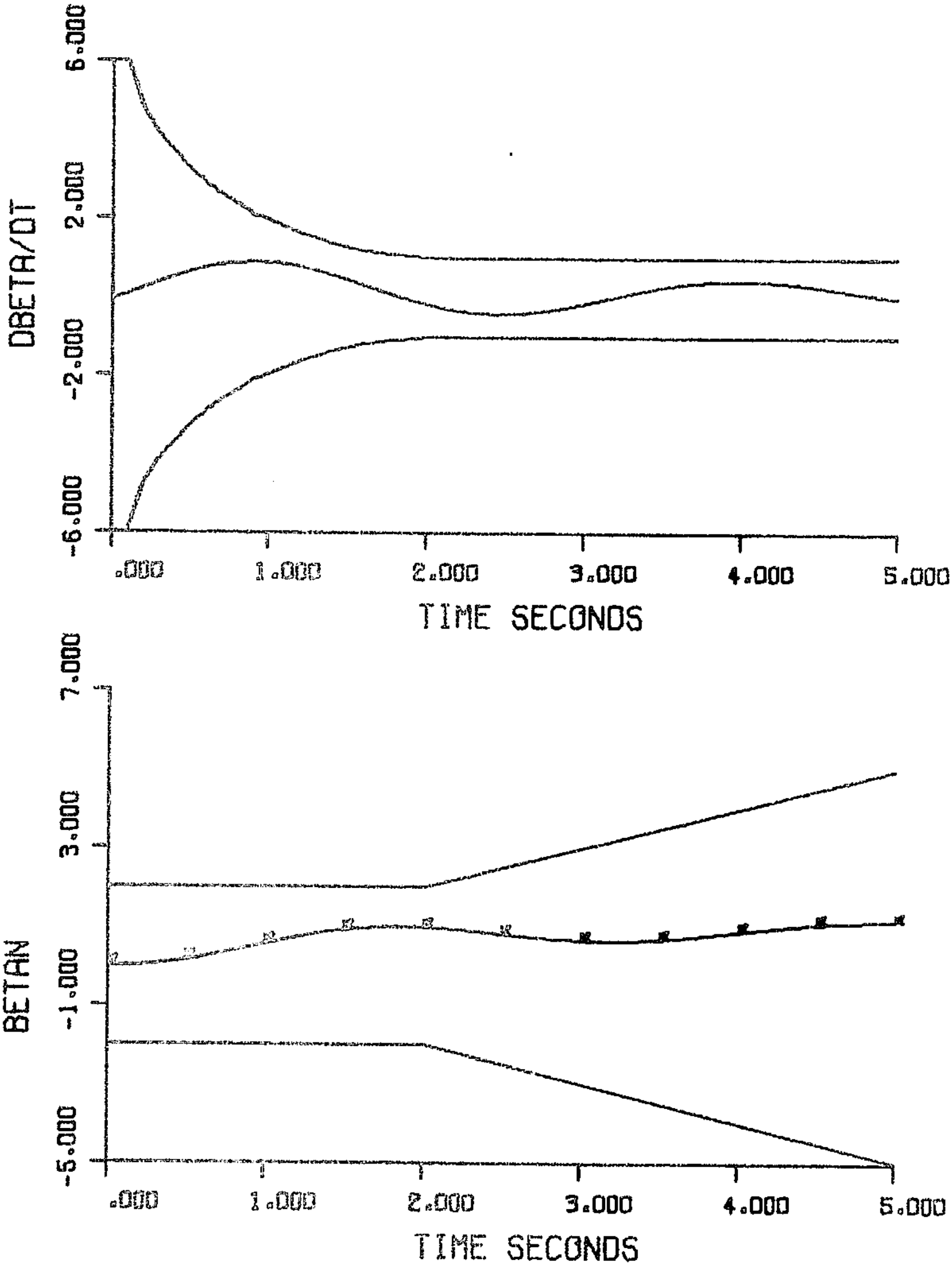


FIG. A3 SIDESLIP

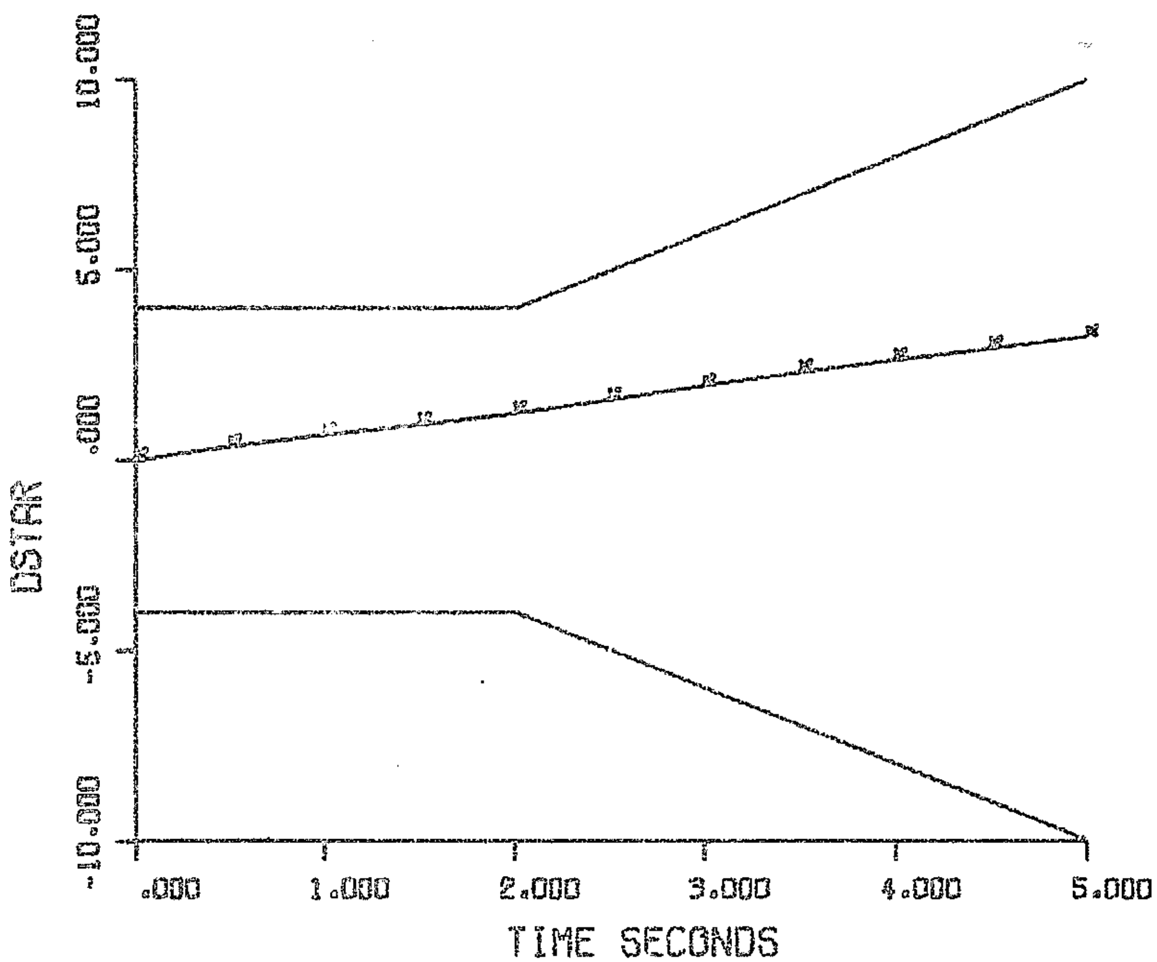


FIG. A4A LATL. CRIT.

FIGURE A-4

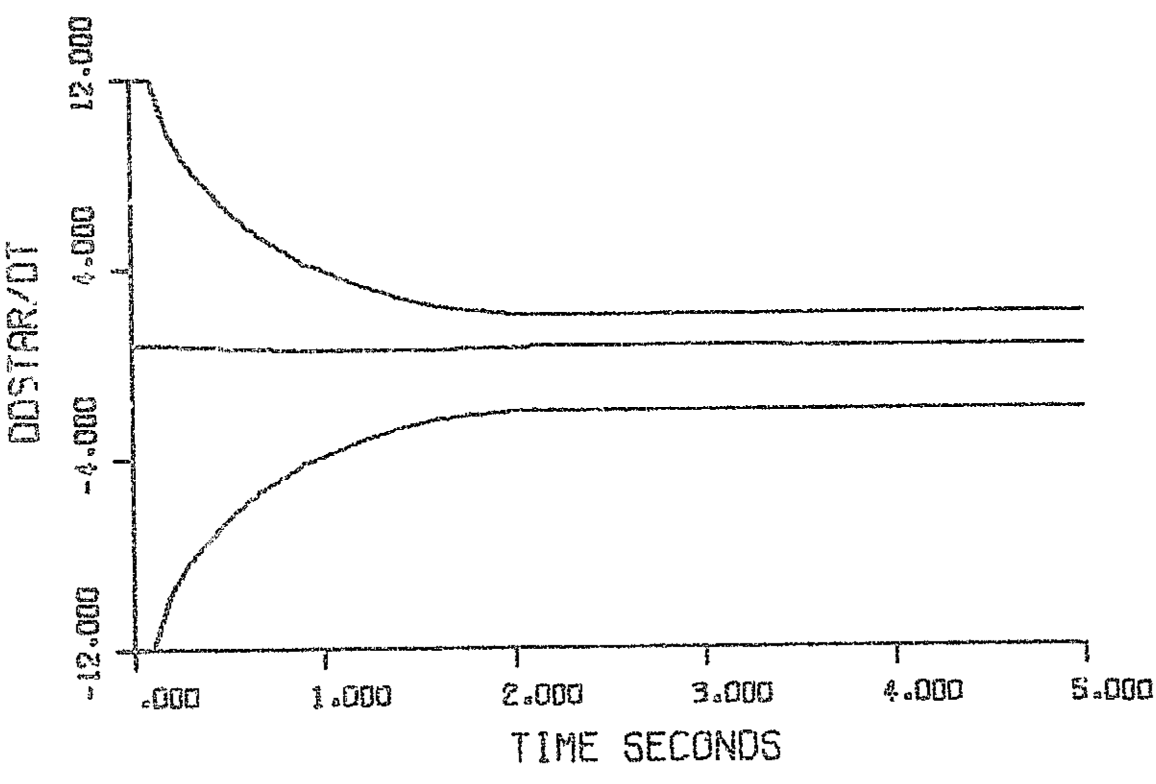


FIG. A4B

EXPERIENCE WITH THE PROGRAM

Several bounds on program flexibility exist. The first involves the generation of eigenvalues from input-time history data. Subroutine MUGEN does calculate a set of λ_i from each set of eleven points representing one input time history. These values are not correct or consistent even when the data is carefully contrived for best results. This is due to the dependence of the λ_i calculation upon differences between adjacent (in time) data values. In anticipation of the program being normally used with data obtained graphically, the values used in the example were held to three significant figures. This so degraded the λ_i calculation that the values from the three input time histories varied from the known values used to generate the data by two orders of magnitude in the case of small eigenvalues and occasionally had the opposite sign. In general, the complex eigenvalues were more closely identified than the real eigenvalues. This was particularly true if the data precision was increased. In view of the anticipated character of the input data and the desirability of specifying the eigenvalues of the resulting model, no further development of MUGEN is planned.

A second difficulty arises when zero suppression is specified. Subroutine CASE2 and CASE3 calculate coefficients, $\underline{C}$, which yield transfer functions having the correct pole-zero excesses. The resulting expression will normally be a poor representation of the input time-history data but that is unavoidable. Unfortunately, the suppression of zeros also tends to ill-condition the numerical equations which must be solved to obtain $\underline{A}$. In extreme cases this prevents convergence of the simple Newton-Euler-Raphson algorithm employed in MODEL.

Finally, there is a general difficulty that may interfere with the integration of the model equations of motion in RESPONS. If the $\underline{A}$ matrix is ill-conditioned an unworkable number of terms in the series approximation for $\underline{e}^{\underline{A}\Delta t}$ may be required. This is caused by the fixed Δt of 0.1 seconds which is built into RESPONS. A limit of 30 terms is imposed. Other limitations will undoubtedly come to light as experience with the program accumulates.

APPENDIX B

Program AANDB Listing

```
      PROGRAM AANDB(INPUT,OUTPUT)                                       MAI
      COMMON /NAMES/NN(20),PN(11),BETAN(11),DSTAR(11),TPN(11)           MAI
     1            ,PNF(51),BETANF(51),DSTARF(51),PN1F(51),BETAN1F(51),DSTMAI
     2AR1F(51),PNC(51),BETANC(51),DSTARC(51),PN1C(51),BETAN1C(51),DSTAR1MAI
     3C(51),APN(11),ABETAN(11),ADSTAR(11),BPN(11),BBETAN(11),BDSTAR(11),MAI
     4PHIN(11)                                                          MAI
      COMMON /PARAM/MU,EIGEN,       C,NCASE,MCASE, NZ,          NPLOT, MAI
     1NHIST,PNFINAL,BNFINAL,DSFINAL,TIME(51),P(4,4),Q(4)                MAI
      COMMON /LIMITS/PNU(51),PNL(51),BETANU(51),BETANL(51),DSTARU(51),  MAI
     1DSTARL(51),PN1U(51),PN1L(51),BETAN1U(51),BETAN1L(51),DSTAR1U(51), MAI
     2DSTAR1L(51)                                                       MAI
      COMMON /FINAL/ PSS,BETASS,DSTARSS,DP,DR,DBETA,DPHI,A(4,4),B(4),DDEMAI
     1LTA                                                               MAI
      COMPLEX MU(4),EIGEN(4),C(5,4)                                     MAI
      DATA PNU/.0,.333,.667,1.0,1.043,1.064,1.076,1.08,1.08,1.078,1.076,MAI
     11.074,1.072,1.07,1.069,1.067,1.065,1.063,1.061,1.059,1.057,1.055,1MAI
     2.053,1.051,1.05,1.048,1.045,1.044,1.042,1.04,1.038,1.036,1.034,1.0MAI
     332,1.03,1.029,1.027,1.025,1.023,1.021,1.019,1.017,1.015,1.013,1.01MAI
     41,1.01,1.008,1.006,1.004,1.002,1.0/,PNL/0.0,0.01,0.027,0.065,0.107MAI
     5,0.16,0.227,0.3,0.36,0.409,0.45,0.479,0.507,.53,0.556,.579,0.6,0.6MAI
     62,0.639,0.656,0.677,0.692,0.708,0.722,0.733,0.748,0.762,0.778,0.79MAI
     7,0.804,0.816,0.829,0.84,0.852,0.861,0.871,0.88,0.888,0.896,0.908,0MAI
     8.915,0.92,0.929,0.938,0.947,0.956,0.965,0.974,0.983,0.992,1.0/    MAI
      DATA BETANU/21*2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2MAI
     1,3.3,3.4,3.5,3.6,3.7,3.8,3.9,4.0,4.1,4.2,4.3,4.4,4.5,4.6,4.7,4.8,4MAI
     2.9,5.0/,BETANL/21*-2.0,-2.1,-2.2,-2.3,-2.4,-2.5,-2.6,-2.7,-2.8,-2.MAI
     39,-3.0,-3.1,-3.2,-3.3,-3.4,-3.5,-3.6,-3.7,-3.8,-3.9,-4.0,-4.1,-4.2MAI
     4,-4.3,-4.4,-4.5,-4.6,-4.7,-4.8,-4.9,-5.0/                         MAI
      DATA DSTARU/21*4.0,4.2,4.4,4.6,4.8,5.0,5.2,5.4,5.6,5.8,6.0,6.2,6.4MAI
     1,6.6,6.8,7.0,7.2,7.4,7.6,7.8,8.0,8.2,8.4,8.6,8.8,9.0,9.2,9.4,9.6,9MAI
     2.8,     10.0/,DSTARL/21*-4.0,-4.2,-4.4,-4.6,-4.8,-5.0,-5.2,-5.4,-5.MAI
     36 ,-5.8,-6.0,-6.2,-6.4,-6.6,-6.8,-7.0,-7.2,-7.4,-7.6,-7.8,-8.0,-8.MAI
     4 2,-8.4,-8.6,-8.8,-9.0,-9.2,-9.4,-9.6,-9.8,-10.0/                 MAI
      DATA PN1U/4.0,4.0,1.686,1.122,0.992,0.908,0.843,0.797,0.724,0.663,MAI
     10.613,0.556,0.510,0.464,0.421,0.379,0.349,0.318,0.295,0.272,0.257,MAI
     20.234,0.222,0.211,0.199,0.192,0.188,0.184,0.180,0.175,0.172,0.168,MAI
     30.165,0.169,0.165,0.165,0.157,0.142,0.119,0.107,0.096,0.080,0.073,MAI
     40.069,0.061,0.054,0.046,0.046,0.046,0.046,0.046/,PN1L/0.806,0.711,MAI
```

```
     50.255,-.224,-.297,-.323,-.327,-.323,-.319,-.304,-.289,-.274,-.251,
     6-.228,-.205,-.183,-.148,-.129,-.125,-.118,-.125,-.123,-.121,-.120,
     7-.118,-.116,-.114,-.112,-.110,-.109,-.107,-.105,-.103,-.102,-.100,
     8-.098,-.096,-.095,-.093,-.091,-.089,-.088,-.086,-.084,-.082,-.081,
     9-.079,-.077,-.075,-.074,-.072/
      DATA BETAN1U/6.0,6.0,4.80,4.15,3.72,3.28,2.91,2.61,2.37,2.10,1.95,
     11.78,1.61,1.49,1.35,1.26,1.17,1.12,1.07,1.05,1.00,30*1./,BETAN1L/
     2-6.0,-6.0,-4.80,-4.15,-3.72,-3.28,-2.91,-2.61,-2.37,-2.10,-1.95,
     3-1.78,-1.61,-1.49,-1.35,-1.26,-1.17,-1.12,-1.07,-1.05,-1.00,30*-1.
     40/
      DATA DSTAR1U/12.0,12.0,9.65,8.30,7.44,6.56,5.82,5.22,4.74,4.20,
     13.90,3.56,3.22,2.98,2.70,2.52,2.34,2.24,2.14,2.10,2.00,30*2.0/,
     2DSTAR1L/-12.0,-12.0,-9.60,-8.30,-7.44,-6.56 ,-5.82,-5.22,-4.74,
     3-4.20,-3.90,-3.56,-3.22,-2.98,-2.70,-2.52,-2.34,-2.24,-2.14,-2.10,
     4-2.00,30*-2.0/
      READ 300,NN
      READ 100,(TPN(I),PN(I),BETAN(I),DSTAR(I),I=1,11)
      READ 200,MCASE,NPLOT,NHIST
      IF(NN(1).EQ.1) CALL LISTER(1)
      CALL MUGEN(PN,MU,EIGEN)
      IF(NN(2).EQ.1) CALL LISTER(2)
      CALL MUGEN(BETAN,MU,EIGEN)
      IF(NN(2).EQ.1) CALL LISTER(2)
      CALL MUGEN(DSTAR,MU,EIGEN)
      IF(NN(2).EQ.1) CALL LISTER(2)
      READ 400,EIGEN
      MU(1)=CEXP(EIGEN(1)*.5)
      MU(2)=CEXP(EIGEN(2)*.5)
      MU(3)=CEXP(EIGEN(3)*.5)
      MU(4)=CONJG(MU(3))
      IF(NN(3).EQ.1) CALL LISTER(3)
      CALL FITTING(PN,TPN,APN)
      DO 10 I=1,11
      PHIN(I)=0.
      DO 10 J=1,11
   10 PHIN(I)=PHIN(I)+APN(J)*  TPN(I)**(12-J)/FLOAT(12-J)
      IF(NN( 4).EQ.1) CALL LISTER(4)
      GO TO (1,2,3,4,5,6,7,8,9),MCASE
    1 CALL CASE1(MU,PN,EIGEN, C(1,1))
      CALL CASE1(MU,BETAN,EIGEN, C(1,2))
```

```
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
    2 CALL CASE3(MU,PN,EIGEN, C(1,1))
      CALL CASE2(MU,BETAN,EIGEN, C(1,2))
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
    3 CALL CASE2(MU,PN,EIGEN, C(1,1))
      CALL CASE3(MU,BETAN,EIGEN, C(1,2))
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
    4 CALL CASE2(MU,PN,EIGEN, C(1,1))
      CALL CASE2(MU,BETAN,EIGEN, C(1,2))
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
    5 CALL CASE2(MU,PN,EIGEN, C(1,1))
      CALL CASE1(MU,BETAN,EIGEN, C(1,2))
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
    6 CALL CASE1(MU,PN,EIGEN, C(1,1))
      CALL CASE2(MU,BETAN,EIGEN, C(1,2))
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
    7 CALL CASE3(MU,PN,EIGEN, C(1,1))
      CALL CASE1(MU,BETAN,EIGEN, C(1,2))
      CALL CASE1(MU,PHIN,EIGEN, C(1,3))
      CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
      IF(NN(5).EQ.1) CALL LISTER(5)
      GO TO 11
```

```
  8 CALL CASE1(MU,PN,EIGEN, C(1,1))
    CALL CASE3(MU,BETAN,EIGEN, C(1,2))
    CALL CASE1(MU,PHIN,EIGEN, C(1,3))
    CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
  9 CALL CASE3(MU,PN,EIGEN, C(1,1))
    CALL CASE3(MU,BETAN,EIGEN, C(1,2))
    CALL CASE1(MU,PHIN,EIGEN, C(1,3))
    CALL CASE1(MU,DSTAR,EIGEN, C(1,4))
    IF(NN(5).EQ.1) CALL LISTER(5)
 11 IF(NN(6).EQ.1) CALL LISTER(6)
    CALL MODEL(C,EIGEN,PSS,BETASS,DSTARSS,DP,DR,DBETA,DPHI,A,B,DDELTA)
    IF(NHIST.NE.0) CALL RESPONS
    IF(NPLOT.NE.0) CALL GRAPHS
    STOP
100 FORMAT(4F10.0)
200 FORMAT(3I4)
300 FORMAT(20I2)
400 FORMAT(8F10.0)
    END
```

```
      SUBROUTINE MUGEN(D,MU,EIGEN)
      DIMENSION D(11) ,P(4,4),Q(4),PI(4,4),A(4)
      COMPLEX MU(4),EIGEN(4),SAVE
      P(1,4)=D(2)-D(1)
      P(1,3)=D(3)-D(2)
      P(1,2)=P(2,4)=D(4)-D(3)
      P(1,1)=P(2,3)=D(5)-D(4)
      P(2,2)=P(3,4)=Q(1)=D(6)-D(5)
      P(2,1)=P(3,3)=D(7)-D(6)
      P(3,2)=P(4,4)=Q(2)=D(8)-D(7)
      P(3,1)=P(4,3)=D(9)-D(8)
      P(4,2)=Q(3)=D(10)-D(9)
      P(4,1)=D(11)-D(10)
      D12=D(9)+3.*(D(11)-D(10))
      Q(4)=D12-D(11)
      CALL INVR(P,PI,4,0,4)
      DO 1 I=1,4
      A(I)=0.
      DO 1 J=1,4
    1 A(I)=A(I)+PI(I,J)*Q(J)
C
C
C     QUADRATIC SYNTHETIC DIVISION
C     NUMERICAL CALCULUS
C     W. E. MILNE    PAGE 53
C
C
      X=0.
      Y=0.
      A0=1.
      A1=-A(1)
      A2=-A(2)
      A3=-A(3)
      A4=-A(4)
   10 B0=A0
      B1=A1-X*B0
      B2=A2-X*B1-Y*B0
      B3=A3-X*B2-Y*B1
      B4=A4-X*B3-Y*B2
```

```
      C0=B0
      C1=B1-X*C0
      C2=B2-X*C1-Y*C0
      C3=  -X*C2-Y*C1
      DD=C2**2-C3*C1
      DX=(B3*C2-B4*C1)/DD
      DY=(B4*C2-B3*C3)/DD
      X=X+DX
      Y=Y+DY
      IF(ABS(DX).GE.0.0001) GO TO 10
      IF(ABS(DY).GE.0.0001) GO TO 10
      U=B1
      V=B2
      ERROR=A4-B2*Y
      IF(ABS(ERROR).LE.0.0001) GO TO 20
      PRINT 100,ERROR
  100 FORMAT(10X,6HERROR=,F10.7)
   20 QUOT=X**2-4.*Y
      IF(QUOT) 21,22,23
   23 ROOT1=-X/2.+SQRT(QUOT)/2.
      ROOT2=-X/2.-SQRT(QUOT)/2.
      MU(1)=CMPLX(ROCT1,0.)
      MU(2)=CMPLX(ROOT2,0.)
      GO TO 24
   22 MU(1)=CMPLX(-X/2.,0.)
      MU(2)=MU(1)
      GO TO 24
   21 MU(1)=CMPLX(-X/2.,SQRT(-QUOT)/2.)
      MU(2)=CMPLX(-X/2.,-SQRT(-QUOT)/2.)
   24 GUOT=U**2-4.*V
      IF(QUOT) 25,26,27
   27 ROOT3=-U/2.+SQRT(QUOT)/2.
      RCOT4=-U/2.-SQRT(QUOT)/2.
      MU(3)=CMPLX(ROCT3,0.)
      MU(4)=CMPLX(ROCT4,0.)
      GO TO 28
   26 MU(3)=CMPLX(-U/2.,0.)
      MU(4)=MU(3)
      GO TO 28
```

```
25 MU(3)=CMPLX(-U/2.,SQRT(-QUOT)/2.)
   MU(4)=CMPLX(-U/2.,-SQRT(-QUOT)/2.)
28 IF(AIMAG(MU(1)).EQ.0.) GO TO 30
   SAVE=MU(1)
   MU(1)=MU(3)
   MU(3)=SAVE
   SAVE=MU(2)
   MU(2)=MU(4)
   MU(4)=SAVE
30 CONTINUE
   DO 40 I=1,4
40 EIGEN(I)=CLOG(MU(I))*CMPLX(2.,0.)
   RETURN
   END
```

```
      SUBROUTINE LISTER(NPRINT)
      COMMON /NAMES/NN(20),PN(11),BETAN(11),DSTAR(11),TPN(11)
     1             ,PNF(51),BETANF(51),DSTARF(51),PN1F(51),BETAN1F(51),DST
     2AR1F(51),PNC(51),BETANC(51),DSTARC(51),PN1C(51),BETAN1C(51),DSTAR1
     3C(51),APN(11),ABETAN(11),ADSTAR(11),BPN(11),BBETAN(11),BDSTAR(11),
     4PHIN(11)
      COMMON /PARAM/MU,EIGEN,         C,NCASE,MCASE, NZ,          NPLOT,
     1NHIST,PNFINAL,BNFINAL,DSFINAL,TIME(51),P(4,4),Q(4)
      COMMON /LIMITS/PNU(51),PNL(51),BETANU(51),BETANL(51),DSTARU(51),
     1DSTARL(51),PN1U(51),PN1L(51),BETAN1U(51),BETAN1L(51),DSTAR1U(51),
     2DSTAR1L(51)
      COMMON /FINAL/ PSS,BETASS,DSTARSS,DP,DR,DBETA,DPHI,A(4,4),B(4),DDE
     1LTA
      COMPLEX MU(4),EIGEN(4),C(5,4)
      PRINT 2000
      GO TO (10,20,30,40,50,60,70,80,90,100,110,120,130,140,150,160,170,
     1180,190,200),NPRINT
   10 CALL DATE(S)
      PRINT 1000,S
      PRINT 1010,NN,MCASE,NPLOT,NHIST,PN,TPN,BETAN,   TPN,DSTAR,   TPN
      GO TO 210
   20 PRINT 1020,MU,EIGEN
      GO TO 210
   30 PRINT 1030,MU,EIGEN
      GO TO 210
   40 PRINT 1040,PHIN,TPN
      GO TO 210
   50 PRINT 1050,C
      GO TO 210
   60 PRINT 1060,MU,EIGEN,C
      GO TO 210
   70 CONTINUE
      GO TO 210
   80 CONTINUE
      GO TO 210
   90 PRINT 1090,(TIME(I),PNU(I),PNL(I),BETANU(I),BETANL(I),DSTARU(I),
     1DSTARL(I),PN1U(I),PN1L(I),BETAN1U(I),BETAN1L(I),DSTAR1U(I),DSTAR1L
     2(I),I=1,51)
      GO TO 210
```

```
100 PRINT 1100,(TIME(I),PNF(I),BETANF(I),DSTARF(I),PN1F(I),BETAN1F(I),
   1DSTAR1F(I),I=1,51)
    GO TO 210
110 PRINT 1110,NZ
    GO TO 210
120 PRINT 1120,((P(I,J),J=1,4),I=1,4)
    GO TO 210
130 PRINT 1130,Q
    GO TO 210
140 PRINT 1140,(TIME(I),PNC(I),BETANC(I),DSTARC(I),I=1,51)
    GO TO 210
150 PRINT 1150,(TIME(I),PN1C(I),BETAN1C(I),DSTAR1C(I),I=1,51)
    GO TO 210
160 PRINT 1160
    GO TO 210
170 PRINT 1170
    GO TO 210
180 PRINT 1180
    GO TO 210
190 CONTINUE
    GO TO 210
200 CONTINUE
210 RETURN
1000 FORMAT(////,10X,                    5HDATE=,A10   ,//)
1010 FORMAT(10X,20HLIST PARAMETER TABLE,20I3//5X12H CASE NUMBER ,I5,10H
    1 PLOT CODE,I5,5X,19H TIME RESPONSE CODE,I5,//,20X,10HINPUT DATA,//
    2,5X,7HPN,    ,11F9.3,/,5X,7HTIME,   ,11F9.2,//,5X,7HBETAN, ,11F9.3,
    3/,5X,7HTIME,   ,11F9.2,//,5X,7HDSTAR, ,11F9.3,/,5X,7HTIME,   ,11F9.2
    4,//)
1020 FORMAT(5X,4(4X,4HREAL,10X,4HIMAG,11X),//,2X,3HMU,,4(2E14.6,5X),//,
    12X,3HEI,,4(F10.4,4X,F10.4,9X))
1030 FORMAT(5X,19HMU VALUES SPECIFIED,//,8X,4(    4HREAL,10X,4HIMAG,10X)
    1,//,1X,3HMU,,4(2E14.6    ),//,1X,6HEIGEN,,4(F10.4,4X,F10.4,4X))
1040 FORMAT(5X,26HTHE GENERATED PHI DATA IS,,//,5X,7HPHIN,   ,11F9.3,
    1/,5X,7HTIME,   ,11F9.2)
1050 FORMAT(5X,26HTIME RESPONSE COEFFICIENTS,//,12X,5(4X,4HREAL,6X,4HIM
    1AG,5X),//,5X,6HPN,    ,5(2F10.4,3X),//,5X,6HBETAN,,5(2F10.4,3X),//,
    25X,6HPHIN, ,5(2F10.4,3X),//,5X,6HDSTAR,,5(2F10.4,3X))
1060 FORMAT(5X,15HFITTING RESULTS,//,5X,5(11X,4HREAL,6X,4HIMAG)    ,//,
```

```
    15X,6HMU,    ,4(2F10.4,5X),//,5X,6HEIGEN,,4(2F10.4,5X),//,5X,
    212HCOEFFICIENTS,//,5X,6HPN,    ,5(2F10.4,5X),//,5X,6HBETAN,,5(2F10.
    34,5X),//,5X,6HPHIN, ,5(2F10.4,5X),//,5X,6HDSTAR,,5(2F10.4,5X))
1070 FORMAT(5X,28HMODEL PARAMETERS-SECOND ROW,,3F10.4,5X,F10.4)
1080 FORMAT(5X,27HMODEL PARAMETERS-THIRD ROW,,3F10.4,5X,F10.4)
1090 FORMAT(5X,18HRESPONSE ENVELOPES,//,8X,4HTIME,12X,2HPN,13X,5HBETAN,
    113X,5HDSTAR,13X,5HPNDOT,10X,8HBETANDOT,10X,8HDSTARDOT,//,51(3X,F9.
    22,12F9.3,/),/)
1100 FORMAT(5X,21HFITTED TIME RESPONSES,//,8X,4HTIME,8X,2HPN,5X,5HBETAN
    1,5X,5HDSTAR,5X,5HPNDOT,2X,8HBETANDOT,2X,8HDSTARDOT,//,51(3X,F9.2,6
    2F10.3,/),/)
1110 FO RMAT(5X,26HPAYNTERS RECIPE NUMBER IS,,I6)
1120 FORMAT(5X,29HDIFFERENCE EQUATION P MATRIX,,4F10.3,/,34X,4F10.3,/,
    134X,4F10.3,/,34X,4F10.3)
1130 FORMAT(5X,29HDIFFERENCE EQUATION G VECTOR,,F10.3,/,34X,F10.3,/,34X
    1,F10.3,/,34X,F10.3)
1140 FORMAT( 5X,19HINTEGRATED RESPONSE,//,8X, 4HTIME,10X,2HPN,7X,5HBETA
    1N,7X,5HDSTAR,//,51(3X,F9.2,3F12.3,/),/)
1150 FORMAT(5X,41HFIRST DERIVATIVES OF INTEGRATED RESPONSES,//,8X,4HTIM
    1E,7X,5HPNDOT,4X,8HBETANDOT,4X,8HDSTARDOT,//,51(3X,F9.2,3F12.3,/),/
    2)
1160 FORMAT(10X,10HPN PLOTTED)
1170 FORMAT(10X,13HBETAN PLOTTED)
1180 FORMAT(10X,13HDSTAR PLOTTED)
2000 FORMAT(///)
     END
```

```
      SUBROUTINE GRAPHS
      COMMON /NAMES/NN(20),PN(11),BETAN(11),DSTAR(11),TPN(11)
     1             ,PNF(51),BETANF(51),DSTARF(51),PN1F(51),BETAN1F(51),DST
     2AR1F(51),PNC(51),BETANC(51),DSTARC(51),PN1C(51),BETAN1C(51),DSTAR1
     3C(51),APN(11),ABETAN(11),ADSTAR(11),BPN(11),BBETAN(11),BDSTAR(11),
     4PHIN(11)
      COMMON /PARAM/MU,EIGEN,          C,NCASE,MCASE, NZ,           NPLOT,
     1NHIST,PNFINAL,BNFINAL,DSFINAL,TIME(51),P(4,4),Q(4)
      COMMON /LIMITS/PNU(51),PNL(51),BETANU(51),BETANL(51),DSTARU(51),
     1DSTARL(51),PN1U(51),PN1L(51),BETAN1U(51),BETAN1L(51),DSTAR1U(51),
     2DSTAR1L(51)
      COMMON /FINAL/ PSS,BETASS,DSTARSS,DP,DR,DBETA,DPHI,A(4,4),B(4),DDE
     1LTA
      COMPLEX MU(4),EIGEN(4),C(5,4)
      IF(NPLOT.EQ.1) GO TO 10
      IF(NPLOT.EQ.3) GO TO 10
      IF(NPLOT.EQ.6) GO TO 10
      IF(NPLOT.NE.7) GO TO 20
   10 IF(NN(16).EQ.1) CALL LISTER(16)
      CALL QIKSET(5.0,0.0,0.0,3.0,0.0,0.5)
      CALL QIKPLT(TIME,PNU,51,14H$TIME SECONDS$,14H$PN 1/SECONDS$,11H$RO
     1LL RATE$)
      CALL PLOT(-6.0,1.0,-3)
      CALL QLINE(TIME,PNL,51,0)
      CALL QLINE(TPN,PN,-11,74)
      CALL QLINE(TIME,PNF,51,0)
      CALL QLINE(TIME,PNC,51,0)
      CALL PLOT(-1.5,3.0,-3)
      CALL QIKSET(5.0,0.0,0.0,3.0,-2.0,2.0)
      CALL QIKPLT(TIME,PN1U,51,14H$TIME SECONDS$,8H$DPN/DT$,3H$ $)
      CALL PLOT(-6.0,1.0,-3)
      CALL QLINE(TIME,PN1L,51,0)
      CALL QLINE(TIME,PN1F,51,0)
      CALL QLINE(TIME,PN1C,51,0)
      CALL ENDPLT
   20 CONTINUE
      IF(NPLOT.EQ.2) GO TO 21
      IF(NPLOT.EQ.3) GO TO 21
      IF(NPLOT.EQ.5) GO TO 21
```

```
      IF(NPLOT.NE.7) GC TO 30
   21 IF(NN(17).EQ.1) CALL LISTER(17)
      CALL QIKSET(5.0,0.0,0.0,3.0,-5.0,4.0)
      CALL QIKPLT(TIME,BETANU,51,14H$TIME SECONDS$,17H$BETAN 1/SECONDS$,
     110H$SIDESLIP$)
      CALL PLOT(-6.0,1.0,-3)
      CALL QLINE(TIME,BETANL,51,0)
      CALL QLINE(TIME,BETANF,51,0)
      CALL QLINE(TPN    ,BETAN,-11,74)
      CALL QLINE(TIME,BETANC,51,0)
      CALL PLOT(-1.5,3.0,-3)
      CALL QIKSET(5.0,0.0,0.0,3.0,-6.0,4.0)
      CALL QIKPLT(TIME,BETAN1U,51,14H$TIME SECONDS$,10H$DBETA/DT$,3H$ $)
      CALL PLOT(-6.0,1.0,-3)
      CALL QLINE(TIME,BETAN1L,51,0)
      CALL QLINE(TIME,BETAN1F,51,0)
      CALL QLINE(TIME,BETAN1C,51,0)
      CALL ENDPLT
   30 CONTINUE
      IF(NPLOT.LT.4) GO TO 40
      IF(NN(18).EQ.1) CALL LISTER(18)
      CALL QIKSET(5.0,0.0,0.0,4.0,-10.0,5.0)
      CALL QIKPLT(TIME,DSTARU,51,14H$TIME SECONDS$,17H$DSTAR 1/SECONDS$,
     113H$LATL. CRIT.$)
      CALL PLOT(-6.0,1.0,-3)
      CALL QLINE(TIME,DSTARL,51,0)
      CALL QLINE(TIME,DSTARF,51,0)
      CALL QLINE(TPN    ,DSTAR,-11,74)
      CALL QLINE(TIME,DSTARC,51,0)
      CALL PLOT(-1.5,4.0,-3)
      CALL QIKSET(5.0,0.0,0.0,3.0,-12.0,8.0)
      CALL QIKPLT(TIME,DSTAR1U,51,14H$TIME SECONDS$,11H$DDSTAR/DT$,3H$ $
     1)
      CALL PLOT(-6.0,1.0,-3)
      CALL QLINE(TIME,DSTAR1L,51,0)
      CALL QLINE(TIME,DSTAR1F,51,0)
      CALL QLINE(TIME,DSTAR1C,51,0)
      CALL ENDPLT
```

```
   40 CONTINUE                                                GRA  780
      RETURN                                                  GRA  790
      END                                                     GRA  800
```

```
      SUBROUTINE CASE1(MU,D,EIGEN,C)
      DIMENSION D(11),U(8,8),UINV(8,8),CC(8),PP(8)
      DIMENSION UU(6,6),UUINV(6,6),PPPP(6),CCC(6)
      DIMENSION UUU(4,4),UUUINV(4,4),PPPPPP(4),CCCC(4)
      COMPLEX MU(4),DD(10),Q(4,4),P(4),RC(10,4),C(5),EIGEN(4)
      COMPLEX QQ(3,3),PPP(3),RRC(10,3)
      COMPLEX QQQ(2,2),PPPPP(2),RRRC(10,2)
      COMPLEX A1,A2,A3,A4
      DO 1 I=1,10
      DO 1 J=1,4
    1 RC(I,J)=(MU(J)**I-1.)
      DO 2 I=1,10
    2 DD(I)=CMPLX((D(I+1)-D(1)),0.)
      DO 3 I=1,4
      DO 3 J=1,4
      Q(I,J)=0.
      DO 3 K=1,10
    3 Q(I,J)=Q(I,J)+RC(K,J)*RC(K,I)
      DO 4 I=1,4
      P(I)=0.
      DO 4 K=1,10
    4 P(I)=P(I)+DD(K)*RC(K,I)
      DO 5 I=1,4
      DO 5 J=1,4
      U(I,J)=REAL(Q(I,J))
      U( I+4 , J+4 )=REAL(Q(I,J))
      U( I+4 ,J)=AIMAG(Q(I,J))
    5 U(I, J+4 )=-AIMAG(Q(I,J))
      CALL INVR(U,UINV,8,0,8)
      DO 6 I=1,4
      PP(I)=REAL(P(I))
    6 PP(I+4)=AIMAG(P(I))
      DO 7 I=1,8
      CC(I)=0.
      DO 7 J=1,8
    7 CC(I)=CC(I)+UINV(I,J)*PP(J)
      DO 8 I=1,4
    8 C(I)=CMPLX(CC(I),CC(I+4))
      C(5)=-C(1)-C(2)-C(3)-C(4)
```

```
      RETURN
      ENTRY CASE2
      A2=-EIGEN(2)/EIGEN(1)
      A3=-EIGEN(3)/EIGEN(1)
      A4=-EIGEN(4)/EIGEN(1)
      DO 10 I=1,10
      RRC(I,1)=1.+A2-MU(2)**I-A2*MU(1)**I
      RRC(I,2)=1.+A3-MU(3)**I-A3*MU(1)**I
   10 RRC(I,3)=1.+A4-MU(4)**I-A4*MU(1)**I
      DO 20 I=1,10
   20 DD(I)=CMPLX(D(I+1),0.)
      DO 30 I=1,3
      DO 30 J=1,3
      QQ(I,J)=0.
      DO 30 K=1,10
   30 QQ(I,J)=QQ(I,J)-RRC(K,J)*RRC(K,I)
      DO 40 I=1,3
      PPP(I)=0.
      DO 40 K=1,10
   40 PPP(I)=PPP(I)+DD(K)*RRC(K,I)
      DO 50 I=1,3
      DO 50 J=1,3
      UU(I,J)=REAL(QQ(I,J))
      UU(I+3,J+3)=REAL(QQ(I,J))
      UU(I+3,J)=AIMAG(QQ(I,J))
   50 UU(I,J+3)=-AIMAG(QQ(I,J))
      CALL INVR(UU,UUINV,6,0,6)
      DO 60 I=1,3
      PPPP(I)=REAL(PPP(I))
   60 PPPP(I+3)=AIMAG(PPP(I))
      DO 70 I=1,6
      CCC(I)=0.
      DO 70 J=1,6
   70 CCC(I)=CCC(I)+UUINV(I,J)*PPPP(J)
      DO 80 I=1,3
   80 C(I+1)=CMPLX(CCC(I),CCC(I+3))
      C(1)=A2*C(2)+A3*C(3)+A4*C(4)
      C(5)=-C(1)-C(2)-C(3)-C(4)
      RETURN
```

```
      ENTRY CASE3
      A1=EIGEN(3)*(EIGEN(3)-EIGEN(2))/(EIGEN(1)*(EIGEN(2)-EIGEN(1)))
      A2=EIGEN(4)*(EIGEN(4)-EIGEN(2))/(EIGEN(1)*(EIGEN(2)-EIGEN(1)))
      A3=EIGEN(3)*(EIGEN(3)-EIGEN(1))/(EIGEN(2)*(EIGEN(1)-EIGEN(2)))
      A4=EIGEN(4)*(EIGEN(4)-EIGEN(1))/(EIGEN(2)*(EIGEN(1)-EIGEN(2)))
      DO 100 I=1,10
      RRRC(I,1)=1.+A3+A1-MU(3)**I-A3*MU(2)**I-A1*MU(1)**I
  100 RRRC(I,2)=1.+A4+A2-MU(4)**I-A4*MU(2)**I-A2*MU(1)**I
      DO 200 I=1,10
  200 DD(I)=CMPLX(D(I+1),0.)
      DO 300 I=1,2
      DO 300 J=1,2
      QQQ(I,J)=0.
      DO 300 K=1,10
  300 QQQ(I,J)=QQQ(I,J)-RRR  (K,J)*RRRC(K,I)
      DO 400 I=1,2
      PPPPP(I)=0.
      DO 400 K=1,10
  400 PPPPP(I)=PPPPP(I)+DD(K)*RRRC(K,I)
      C(3)=(QQQ(2,2)*PPPPP(1)-QQQ(1,2)*PPPPP(2))/(QQQ(1,1)*QQQ(2,2)-
     1QQQ(1,2)*QQQ(2,1))
      C(4)=(QQQ(1,1)*PPPPP(2)-QQQ(2,1)*PPPPP(1))/(QQQ(1,1)*QQQ(2,2)-
     1QQQ(1,2)*QQQ(2,1))
      C(1)=A1*C(3)+A2*C(4)
      C(2)=A3*C(3)+A4*C(4)
      C(5)=-C(1)-C(2)-C(3)-C(4)
      RETURN
      END
```

```
      SUBROUTINE INVR(A,B,JJJ,IT,MX)
C    1MX,MY,MU,MS,MAT1,MAT2,MAT3,MAT4,MAT5,MAT6)
C   PROGRAM AUTHORS R.E. FUNDERIC AND R.G. EDWARDS,
C   COMPUTINF TECHNOLOGY CENTER, UNION CARBIDE CORP., NUCLEAR DIV.,
C   OAK RIDGE, TENN.
C
C   CTC ORD PROGRAM NO. 9067.1
      DIMENSION A(MX,MX),B(MX,MX)
      MAT1=MX
      MAT2=MX
      IF (JJJ.NE.1) GO TO 50
      B(1,1)=1./A(1,1)
      RETURN
   50 CONTINUE
      DO 21 I=1,JJJ
      DO 20 J=1,JJJ
      B(I,J)=0.0
   20 CONTINUE
      B(I,I)=1.0
   21 CONTINUE
      KK=JJJ
      NV=JJJ
      D=1.
      IF (JJJ.LT.0)D=0.
      KKM=KK-1
      DO 9 I=1,KKM
      S=0.0
      DO 1 J=I,KK
      R=ABS(A(J,I))
      IF (R.LT.S) GO TO 1
      S=R
      L=J
    1 CONTINUE
      IF (L.EQ.I) GO TO 5
      DO 2 J=I,KK
      S=A(I,J)
      A(I,J)=A(L,J)
      A(L,J)=S
    2 CONTINUE
```

```
      IF(NV.LE.0)GO TO 4
      DO 3 J=1,NV
      S=B(I,J)
      B(I,J)=B(L,J)
      B(L,J)=S
    3 CONTINUE
    4 D=-D
    5 IF(A(I,I).EQ.0.)GO TO 9
      IPO=I+1
      DO 8 J=IPO,KK
      IF (A(J,I).EQ.0.) GO TO 8
      S=A(J,I)/A(I,I)
      A(J,I)=0.0
      DO 6 K=IPO,KK
      A(J,K)=A(J,K)-A(I,K)*S
    6 CONTINUE
      IF (NV.LE.0) GO TO 8
      DO 7 K=1,NV
      B(J,K)=B(J,K)-B(I,K)*S
    7 CONTINUE
    8 CONTINUE
    9 CONTINUE
      DO 10 I=1,KK
      D=D*A(I,I)
   10 CONTINUE
      IF(NV.LE.0)GO TO 13
      KMO=KK-1
      DO 12 K=1,NV
      B(KK,K)=B(KK,K)/A(KK,KK)
      DO 12 I=1,KMO
      N=KK-I
      DO 11 J=N,KMO
      B(N,K)=B(N,K)-A(N,J+1)*B(J+1,K)
   11 CONTINUE
      B(N,K)=B(N,K)/A(N,N)
   12 CONTINUE
   13 DMATEQ=D
      IF (IT.EQ.0) RETURN
      DO 30 I=1,JJJ
      DO 30 J=1,JJJ
```

```
      IF (ABS(B(I,J)).LT.1.E-5)B(I,J)=0.
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RESPONS
      COMMON /NAMES/NN(20),PN(11),BETAN(11),DSTAR(11),TPN(11)
     1            ,PNF(51),BETANF(51),DSTARF(51),PN1F(51),BETAN1F(51),DST
     2AR1F(51),PNC(51),BETANC(51),DSTARC(51),PN1C(51),BETAN1C(51),DSTAR1
     3C(51),APN(11),ABETAN(11),ADSTAR(11),BPN(11),BBETAN(11),BDSTAR(11),
     4PHIN(11)
      COMMON /PARAM/MU,EIGEN,        C,NCASE,MCASE, NZ,          NPLOT,
     1NHIST,PNFINAL,BNFINAL,DSFINAL,TIME(51),P(4,4),Q(4)
      COMMON /LIMITS/PNU(51),PNL(51),BETANU(51),BETANL(51),DSTARU(51),
     1DSTARL(51),PN1U(51),PN1L(51),BETAN1U(51),BETAN1L(51),DSTAR1U(51),
     2DSTAR1L(51)
      COMMON /FINAL/ PSS,BETASS,DSTARSS,DP,DR,DBETA,DPHI,A(4,4),B(4),DDE
     1LTA
      COMPLEX MU(4),EIGEN(4),C(5,4)
      DIMENSION TEMP(4,4),XTEMP(4,4),YTEMP(4,4),R(4,4)
      DIMENSION XR1(51),XR2(51),XR3(51),XR4(51)
      DO 1 I=1,51
    1 TIME(I)=FLOAT(I-1)/10.
      IF(NN(9).EQ.1) CALL LISTER(9)
      DO 10 I=1,51
      PNF(I)=C(1,1)*CEXP(EIGEN(1)*TIME(I))+C(2,1)*CEXP(EIGEN(2)*TIME(I))
     1+C(3,1)*CEXP(EIGEN(3)*TIME(I))+C(4,1)*CEXP(EIGEN(4)*TIME(I))+C(5,1
     2)
      BETANF(I)=C(1,2)*CEXP(EIGEN(1)*TIME(I) )+C(2,2)*CEXP(EIGEN(2)*TIME
     1(I))+C(3,2)*CEXP(EIGEN(3)*TIME(I))+C(4,2)*CEXP(EIGEN(4)*TIME(I))+
     2C(5,2)
      DSTARF(I)=C(1,4)*CEXP(EIGEN(1)*TIME(I) )+C(2,4)*CEXP(EIGEN(2)*TIME
     1(I))+C(3,4)*CEXP(EIGEN(3)*TIME(I))+C(4,4)*CEXP(EIGEN(4)*TIME(I))+
     2C(5,4)
      PN1F(I)=EIGEN(1)*C(1,1)*CEXP(EIGEN(1)*TIME(I)) +EIGEN(2)*C(2,1)
     1*CEXP(EIGEN(2)*TIME(I))+EIGEN(3)*C(3,1)*CEXP(EIGEN(3)*TIME(I))+
     2EIGEN(4)*C(4,1)*CEXP(EIGEN(4)*TIME(I))
      BETAN1F(I)=EIGEN(1)*C( 1,2)*CEXP(EIGEN(1)*TIME(I))+EIGEN(2)*C(2,2)
     1*CEXP(EIGEN(2)*TIME(I))+EIGEN(3)*C(3,2)*CEXP(EIGEN(3)*TIME(I))+
     2EIGEN(4)*C(4,2)*CEXP(EIGEN(4)*TIME(I))
      DSTAR1F(I)=EIGEN(1)*C(1,4)*CEXP(EIGEN(1)*TIME(I))+EIGEN(2)*C(2,4)*
     1CEXP(EIGEN(2)*TIME(I))+EIGEN(3)*C(3,4)*CEXP(EIGEN(3)*TIME(I))+
     2EIGEN(4)*C(4,4)*CEXP(EIGEN(4)*TIME(I))
   10 CONTINUE
```

```
      IF(NN(10).EQ.1) CALL LISTER(10)
C
C     PAYNTERS RECIPE
C
      AA=ABS(A(1))
      DO 20 J=2,16
      IF(ABS(A(J)).GT.ABS(AA     )) AA=ABS(A(J))
   20 CONTINUE
      AAT=0.1*AA
      FACT=1.
      DO 30 K=1,30
      FACT=FACT*FLOAT(K)
      X=(4.*AAT)**K*EXP(4.*AAT)/FACT
      IF(X.LT.0.001) GO TO 40
   30 NZ=K+1
   40 CONTINUE
      IF(NN(11).EQ.1) CALL LISTER(11)
      TEMP(1)=TEMP(6)=TEMP(11)=TEMP(16)=1.
      TEMP(2)=TEMP(3)=TEMP(4)=TEMP(5)=TEMP(7)=TEMP(8)=TEMP(9)=TEMP(10)=0
      TEMP(12)=TEMP(13)=TEMP(14)=TEMP(15)=0.
      P(1,1)=P(2,2)=P(3,3)=P(4,4)=1.
      P(1,2)=P(1,3)=P(1,4)=P(2,1)=P(2,3)=P(2,4)=P(3,1)=P(3,2)=P(3,4)=0.
      P(4,1)=P(4,2)=P(4,3)=0.
      DO 50 I=1,NZ
      DO 51 JJ=1,16
   51 XTEMP(JJ)=0.0
      DO 55 J=1,4
      DO 55 K=1,4
      DO 55 L=1,4
   55 XTEMP(J,K)=XTEMP(J,K)+TEMP(J,L)*A(L,K)*0.1/FLOAT(I)
      DO 52 JJ=1,16
   52 TEMP(JJ)=XTEMP(JJ)
      DO 60 II=1,16
   60 P(II)=P(II)+TEMP(II)
   50 CONTINUE
      IF(NN(12).EQ.1) CALL LISTER(12)
      TEMP(1)=TEMP(6)=TEMP(11)=TEMP(16)=1.
      TEMP(2)=TEMP(3)=TEMP(4)=TEMP(5)=TEMP(7)=TEMP(8)=TEMP(9)=TEMP(10)=0
      TEMP(12)=TEMP(13)=TEMP(14)=TEMP(15)=0.
```

C.2

```
      R(1,1)=R(2,2)=R(3,3) =R(4,4)=1.
      R(1,2)=R(1,3)=R(1,4)=R(2,1)=R(2,3)=R(2,4)=R(3,1)=R(3,2)=R(3,4)=0.
      R(4,1)=R(4,2)=R(4,3)=0.
      DO 61 I=1,NZ
      DO 62 JJ=1,16
   62 XTEMP(JJ)=0.0
      DO 65 J=1,4
      DO 65 K=1,4
      DO 65 L=1,4
   65 XTEMP(J,K)=XTEMP(J,K)+TEMP(J,L)*A(L,K)*0.1/FLOAT(I+1)
      DO 67 JJ=1,16
   67 TEMP(JJ)=XTEMP(JJ)
      DO 69 II=1,16
   69 R(II)=R(II)+TEMP(II)
   61 CONTINUE
      Q(1)=R(1,1)*B(1)+R(1,2)*B(2)+R(1,3)*B(3)+R(1,4)*B(4)
      Q(2)=R(2,1)*B(1)+R(2,2)*B(2)+R(2,3)*B(3)+R(2,4)*B(4)
      Q(3)=R(3,1)*B(1)+R(3,2)*B(2)+R(3,3)*B(3)+R(3,4)*B(4)
      Q(4)=R(4,1)*B(1)+R(4,2)*B(2)+R(4,3)*B(3)+R(4,4)*B(4)
      Q(1)=Q(1)*0.1
      Q(2)=Q(2)*0.1
      Q(3)=Q(3)*0.1
      Q(4)=Q(4)*0.1
      IF(NN(13).EQ.1) CALL LISTER(13)
      XR1(1)        =XR3(1)=XR4(1)=0.
      XR2(1)=-DDELTA/DR
      DO 70 I=2,51
      XR1(I)=P(1,1)*XR1(I-1)+P(1,2)*XR2(I-1)+P(1,3)*XR3(I-1)+P(1,4)*XR4(
     1I-1)+Q(1)
      XR2(I)=P(2,1)*XR1(I-1)+P(2,2)*XR2(I-1)+P(2,3)*XR3(I-1)+P(2,4)*XR4(
     1I-1)+Q(2)
      XR3(I)=P(3,1)*XR1(I-1)+P(3,2)*XR2(I-1)+P(3,3)*XR3(I-1)+P(3,4)*XR4(
     1I-1)+Q(3)
   70 XR4(I)=P(4,1)*XR1(I-1)+P(4,2)*XR2(I-1)+P(4,3)*XR3(I-1)+P(4,4)*XR4(
     1I-1)+Q(4)
      DO 71 I=1,51
      PNC(I)=PSS*XR1(I)
      BETANC(I)=BETASS*XR3(I)
   71 DSTARC(I)=DP*XR1(I)+DR*XR2(I)+DBETA*XR3(I)+DPHI*XR4(I)+DDELTA
```

```
      IF(NN(14).EQ.1) CALL LISTER(14)
      DO 80 I=1,51
      XR1T=XR1(I)*A(1,1)+XR2(I)*A(1,2)+XR3(I)*A(1,3)+XR4(I)*A(1,4)+B(1)
      XR2T=XR1(I)*A(2,1)+XR2(I)*A(2,2)+XR3(I)*A(2,3)+XR4(I)*A(2,4)+B(2)
      XR3T=XR1(I)*A(3,1)+XR2(I)*A(3,2)+XR3(I)*A(3,3)+XR4(I)*A(3,4)+B(3)
      XR4T=XR1(I)*A(4,1)+XR2(I)*A(4,2)+XR3(I)*A(4,3)+XR4(I)*A(4,4)+B(4)
      PN1C(I)=PSS*XR1T
      BETAN1C(I)=BETASS*XR3T
   80 DSTAR1C(I)=DP*XR1T+DR*XR2T+DBETA*XR3T+DPHI*XR4T
      IF(NN(15).EQ.1) CALL LISTER(15)
      RETURN
      END
```

```
      SUBROUTINE MODEL(C,EIGEN,PSS,BETASS,DSS     ,DP,DR,DBETA,DPHI,A,B,
     1DDELTA)
      COMPLEX C(5,4),EIGEN(4)
      DIMENSION A(4,4),B(4),X(16),XINC(16),P(16,16),PINV(16,16),F(16),
     1T(4,4),AT(4,4),W(4)
      REAL L,L1,L2,K,I31,I32,I33,I34
      DATA X/1.,4*0.,1.,4*0.,1.,4*0.,1./
      READ 100,V,L,C3,QCO,PSS,BETASS,DSS
      PRINT 200,V,L,C3,QCO,PSS,BETASS,DSS
      READ 110,ITMAX,EPS1
      PRINT 300,ITMAX,EPS1,X
      K=C3*QCO
      L1=REAL(EIGEN(1))
      L2=REAL(EIGEN(2))
      A3=REAL(EIGEN(3))
      B3=AIMAG(EIGEN(3))
      R11=REAL(C(1,1))/PSS
      R12=REAL(C(1,2))/BETASS
      R13=REAL(C(1,3))/PSS
      R14=REAL(C(1,4))/DSS
      R21=REAL(C(2,1))/PSS
      R22=REAL(C(2,2))/BETASS
      R23=REAL(C(2,3))/PSS
      R24=REAL(C(2,4))/DSS
      R31=REAL(C(3,1))/PSS
      R32=REAL(C(3,2))/BETASS
      R33=REAL(C(3,3))/PSS
      R34=REAL(C(3,4))/DSS
      I31=AIMAG(C(3,1))/PSS
      I32=AIMAG(C(3,2))/BETASS
      I33=AIMAG(C(3,3))/PSS
      I34=AIMAG(C(3,4))/DSS
      C1=V*R11
      C2=C1*L/V
      C3=V*R12
      C4=C3*L/V
      C5=V*R13
      C6=C5*L/V
      C7=(R14-K*R12)
```

```
C8=-V*R11*L1
C9=C8*L/V
D1=-C8
C10=V*R21
C11=C10*L/V
C12=V*R22
C13=C12*L/V
C14=V*R23
C15=C14*L/V
C16=(R24-K*R22)
C17=-V*R21*L2
C18=C17*L/V
D2=-C17
C19=V*R31
C20=C19*L/V
C21=V*R32
C22=C21*L/V
C23=V*R33
C24=C23*L/V
C25=R34-K*R32
C26=V*(I31*B3-R31*A3)
C27=C26*L/V
D3=-C26
C28=V*I31
C29=C28*L/V
C30=V*I32
C31=C30*L/V
C32=V*I33
C33=C32*L/V
C34=I34-K*I32
C35=-V*(R31*B3+I31*A3)
C36=C35*L/V
D4=-C35
C37=V*R11+L*R11*L1
C38=V*R12+L*R12*L1
C39=V*R13+L*R13*L1
C40=V*R11*L1
C41=V*R12*L1
C42=V*R13*L1
```

```
D5=R14*L1-K*R12*L1
C43=V*R21+L*R21*L2
C44=V*R22+L*R22*L2
C45=V*R23+L*R23*L2
C46=V*R21*L2
C47=V*R22*L2
C48=V*R23*L2
D6=R24*L2-K*R22*L2
C49=V*R31+L*R31*A3-L*I31*B3
C50=V*R32+L*R32*A3-L*I32*B3
C51=V*R33+L*R33*A3-L*I33*B3
C52=V*R31*A3-V*I31*B3
C53=V*(R32*A3-I32*B3)
C54=V*(R33*A3-I33*B3)
D7= R34*A3-I34*B3-K*R32*A3+K*I32*B3
C55=V*I31+L*I31*A3+L*R31*B3
C56=V*I32+L*R32*B3+L*I32*A3
C57=V*I33+L*R33*B3+L*I33*A3
C58=V*R31*B3+V*I31*A3
C59=V*(R32*B3+I32*A3)
C60=V*(R33*B3+I33*A3)
D8=R34*B3+I34*A3-K*R32*B3-K*I32*A3
C61=R14-K*R12-V*R12*L1
C62=-L*R12*L1
D9=-C62*V/L
C63=R24-K*R22-V*L2*R22
C64=-L*L2*R22
D10=-C64*V/L
C65=R34-K*R32-V*R32*A3+V*I32*B3
C66=L*(-R32*A3+I32*B3)
D11=-C66*V/L
C67=I34-K*I32-V*I32*A3-V*R32*B3
C68=-L*(R32*B3+I32*A3)
D12=-C68*V/L
C69=-V*R13*L1
C70=C69*L/V
D13=-C69
C71=-V*R23*L2
C72=C71*L/V
D14=-C71
```

```
      C73=-V*(R33*A3-I33*B3)
      C74=C73*L/V
      D15=-C73
      C75=-V*(R33*B3+I33*A3)
      C76=C75*L/V
      D16=-C75
      DO 4 ITER=1,ITMAX
      AA=X(1)*X(10)-X(2)*X(9)
      AB=X(1)*X(6)-X(2)*X(5)
      AC=X(3)*X(10)-X(2)*X(11)
      AD=X(3)*X(6)-X(2)*X(7)
      AE=X(4)*X(10)-X(2)*X(12)
      AF=X(4)*X(6)-X(2)*X(8)
      F(1)=C1*AA+C2*AB+C3*AC+C4*AD+C5*AE+C6*AF+C1*X(1)+C7*X(2)+C3*X(3)+
     1C5*X(4)+C8*X(10)+C9*X(6)-D1
      F(2)=C10*AA+C11*AB+C12*AC+C13*AD+C14*AE+C15*AF+C10*X(1)+C16*X(2)+
     1C12*X(3)+C14*X(4)+C17*X(10)+C18*X(6)-D2
      F(3)=C19*AA+C20*AB+C21*AC+C22*AD+C23*AE+C24*AF+C19*X(1)+C25*X(2)+
     1C21*X(3)+C23*X(4)+C26*X(10)+C27*X(6)-D3
      F(4)=C28*AA+C29*AB+C30*AC+C31*AD+C32*AE+C33*AF+C28*X(1)+C34*X(2)+
     1C30*X(3)+C32*X(4)+C35*X(10)+C36*X(6)-D4
      AA=X(5)*X(10)-X(6)*X(9)
      AB=X(7)*X(10)-X(6)*X(11)
      AC=X(8)*X(10)-X(6)*X(12)
      F(5)=C1*AA+C3*AB+C5*AC+C37*X(5)+C7*X(6)+C38*X(7)+C39*X(8)+C40*X(9)
     1+C41*X(11)+C42*X(12)-D5
      F(6)=C10*AA+C12*AB+C14*AC+C43*X(5)+C16*X(6)+C44*X(7)+C45*X(8)+C46*
     1X(9)+C47*X(11)+C48*X(12)-D6
      F(7)=C19*AA+C21*AB+C23*AC+C49*X(5)+C25*X(6)+C50*X(7)+C51*X(8)+C52*
     1X(9)+C53*X(11)+C54*X(12)-D7
      F(8)=C28*AA+C30*AB+C32*AC+C55*X(5)+C34*X(6)+C56*X(7)+C57*X(8)+C58*
     1X(9)+C59*X(11)+C60*X(12)-D8
      AA=X(9)*X(6)-X(5)*X(10)
      AB=X(11)*X(6)-X(7)*X(10)
      AC=X(12)*X(6)-X(8)*X(10)
      F(9)=C2*AA+C4*AB+C6*AC+C1*X(9)+C61*X(10)+C3*X(11)+C5*X(12)+C62*
     1X(6)-D9
      F(10)=C11*AA+C13*AB+C15*AC+C10*X(9)+C63*X(10)+C12*X(11)+C14*X(12)+
     1C64*X(6)-D10
```

```
      F(11)=C20*AA+C22*AB+C24*AC+C19*X(9)+C65*X(10)+C21*X(11)+C23*X(12)+MOD 1580
     1C66*X(6)-D11                                                      MOD 1590
      F(12)=C29*AA+C31*AB+C33*AC+C28*X(9)+C67*X(10)+C30*X(11)+C32*X(12)+MOD 1600
     1C68*X(6)-D12                                                      MOD 1610
      AA=X(13)*X(10)-X(14)*X(9)                                         MOD 1620
      AB=X(13)*X(6)-X(14)*X(5)                                          MOD 1630
      AC=X(15)*X(10)-X(14)*X(11)                                        MOD 1640
      AD=X(15)*X(6)-X(14)*X(7)                                          MOD 1650
      AE=X(16)*X(10)-X(14)*X(12)                                        MOD 1660
      AF=X(16)*X(6)-X(14)*X(8)                                          MOD 1670
      F(13)=C1*AA+C2*AB+C3*AC+C4*AD+C5*AE+C6*AF+C1*X(13)+C7*X(14)+C3*   MOD 1680
     1X(15)+C5*X(16)+C69*X(10)+C70*X(6)-D13                             MOD 1690
      F(14)=C10*AA+C11*AB+C12*AC+C13*AD+C14*AE+C15*AF+C10*X(13)+C16*    MOD 1700
     1X(14)+C12*X(15)+C14*X(16)+C71*X(10)+C72*X(6)-D14                  MOD 1710
      F(15)=C19*AA+C20*AB+C21*AC+C22*AD+C23*AE+C24*AF+C19*X(13)+C25*    MOD 1720
     1X(14)+C21*X(15)+C23*X(16)+C73*X(10)+C74*X(6)-D15                  MOD 1730
      F(16)=C28*AA+C29*AB+C30*AC+C31*AD+C32*AE+C33*AF+C28*X(13)+C34*    MOD 1740
     1X(14)+C30*X(15)+C32*X(16)+C75*X(10)+C76*X(6)-D16                  MOD 1750
      DO 5 I=1,256                                                      MOD 1760
    5 P(I)=0.                                                           MOD 1770
      P(1,1)=C1*X(10)+C2*X(6)+C1                                        MOD 1780
      P(1,2)=-C1*X(9)-C2*X(5)-C3*X(11)-C4*X(7)-C5*X(12)-C6*X(8)+C7      MOD 1790
      P(1,3)=C3*X(10)+C4*X(6)+C3                                        MOD 1800
      P(1,4)=C5*X(10)+C6*X(6)+C5                                        MOD 1810
      P(1,5)=-C2*X(2)                                                   MOD 1820
      P(1,6)=C2*X(1)+C4*X(3)+C6*X(4)+C9                                 MOD 1830
      P(1,7)=-C4*X(2)                                                   MOD 1840
      P(1,8)=-C6*X(2)                                                   MOD 1850
      P(1,9)=-C1*X(2)                                                   MOD 1860
      P(1,10)=C1*X(1)+C3*X(3)+C5*X(4)+C8                                MOD 1870
      P(1,11)=-C3*X(2)                                                  MOD 1880
      P(1,12)=-C5*X(2)                                                  MOD 1890
      P(2,1)=C10*X(10)+C11*X(6)+C10                                     MOD 1900
      P(2,2)=-C10*X(9)-C11*X(5)-C12*X(11)-C13*X(7)-C14*X(12)-C15*X(8)+  MOD 1910
     1C16                                                               MOD 1920
      P(2,3)=C12*X(10)+C13*X(6)+C12                                     MOD 1930
      P(2,4)=C14*X(10)+C15*X(6)+C14                                     MOD 1940
      P(2,5)=-C11*X(2)                                                  MOD 1950
      P(2,6)=C11*X(1)+C13*X(3)+C15*X(4)+C18                             MOD 1960
      P(2,7)=-C13*X(2)                                                  MOD 1970
```

```
      P(2,8)=-C15*X(2)
      P(2,9)=-C10*X(2)
      P(2,10)=C10*X(1)+C12*X(3)+C14*X(4)+C17
      P(2,11)=-C12*X(2)
      P(2,12)=-C14*X(2)
      P(3,1)=C19*X(10)+C20*X(6)+C19
      P(3,2)=-C19*X(9)-C20*X(5)-C21*X(11)-C22*X(7)-C23*X(12)-C24*X(8)+
     1C25
      P(3,3)=C21*X(10)+C22*X(6)+C21
      P(3,4)=C23*X(10)+C24*X(6)+C23
      P(3,5)=-C20*X(2)
      P(3,6)=C20*X(1)+C22*X(3)+C24*X(4)+C27
      P(3,7)=-C22*X(2)
      P(3,8)=-C24*X(2)
      P(3,9)=-C19*X(2)
      P(3,10)=C19*X(1)+C21*X(3)+C23*X(4)+C26
      P(3,11)=-C21*X(2)
      P(3,12)=-C23*X(2)
      P(4,1)=C28*X(10)+C29*X(6)+C28
      P(4,2)=-C28*X(9)-C29*X(5)-C30*X(11)-C31*X(7)-C32*X(12)-C33*X(8)+
     1C34
      P(4,3)=C30*X(10)+C31*X(6)+C30
      P(4,4)=C32*X(10)+C33*X(6)+C32
      P(4,5)=-C29*X(2)
      P(4,6)=C29*X(1)+C31*X(3)+C33*X(4)+C36
      P(4,7)=-C31*X(2)
      P(4,8)=-C33*X(2)
      P(4,9)=-C28*X(2)
      P(4,10)=C28*X(1)+C30*X(3)+C32*X(4)+C35
      P(4,11)=-C30*X(2)
      P(4,12)=-C32*X(2)
      P(5,5)=C1*X(10)+C37
      P(5,6)=-C1*X(9)-C3*X(11)-C5*X(12)+C7
      P(5,7)=C3*X(10)+C38
      P(5,8)=C5*X(10)+C39
      P(5,9)=-C1*X(6)+C40
      P(5,10)=C1*X(5)+C3*X(7)+C5*X(8)
      P(5,11)=-C3*X(6)+C41
      P(5,12)=-C5*X(6)+C42
      P(6,5)=C10*X(10)+C43
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P(6,6)=-C10*X(9)-C12*X(11)-C14*X(12)+C16          MOD 2380
P(6,7)=C12*X(10)+C44                              MOD 2390
P(6,8)=C14*X(10)+C45                              MOD 2400
P(6,9)=-C10*X(6)+C46                              MOD 2410
P(6,10)=C10*X(5)+C12*X(7)+C14*X(8)                MOD 2420
P(6,11)=-C12*X(6)+C47                             MOD 2430
P(6,12)=-C14*X(6)+C48                             MOD 2440
P(7,5)=C19*X(10)+C49                              MOD 2450
P(7,6)=-C19*X(9)-C21*X(11)-C23*X(12)+C25          MOD 2460
P(7,7)=C21*X(10)+C50                              MOD 2470
P(7,8)=C23*X(10)+C51                              MOD 2480
P(7,9)=-C19*X(6)+C52                              MOD 2490
P(7,10)=C19*X(5)+C21*X(7)+C23*X(8)                MOD 2500
P(7,11)=-C21*X(6)+C53                             MOD 2510
P(7,12)=-C23*X(6)+C54                             MOD 2520
P(8,5)=C28*X(10)+C55                              MOD 2530
P(8,6)=-C28*X(9)-C30*X(11)-C32*X(12)+C34          MOD 2540
P(8,7)=C30*X(10)+C56                              MOD 2550
P(8,8)=C32*X(10)+C57                              MOD 2560
P(8,9)=-C28*X(6)+C58                              MOD 2570
P(8,10)=C28*X(5)+C30*X(7)+C32*X(8)                MOD 2580
P(8,11)=-C30*X(6)+C59                             MOD 2590
P(8,12)=-C32*X(6)+C60                             MOD 2600
P(9,6)=C2*X(9)+C4*X(11)+C6*X(12)+C62              MOD 2610
P(9,5)=-C2*X(10)                                  MOD 2620
P(9,7)=-C4*X(10)                                  MOD 2630
P(9,8)=-C6*X(10)                                  MOD 2640
P(9,9)=C2*X(6)+C1                                 MOD 2650
P(9,10)=-C2*X(5)-C4*X(7)-C6*X(8)+C61              MOD 2660
P(9,11)=C4*X(6)+C3                                MOD 2670
P(9,12)=C6*X(6)+C5                                MOD 2680
P(10,5)=-C11*X(10)                                MOD 2690
P(10,6)=C11*X(9)+C13*X(11)+C15*X(12)+C64          MOD 2700
P(10,8)=-C15*X(10)                                MOD 2710
P(10,7)=-C13*X(10)                                MOD 2720
P(10,9)=C11*X(6)+C10                              MOD 2730
P(10,10)=-C11*X(5)-C13*X(7)-C15*X(8)+C63          MOD 2740
P(10,11)=C13*X(6)+C12                             MOD 2750
P(10,12)=C15*X(6)+C14                             MOD 2760
```

```
P(11,5)=-C20*X(10)                                                      MOD 2770
P(11,6)=C20*X(9)+C22*X(11)+C24*X(12)+C66                                MOD 2780
P(11,7)=-C22*X(10)                                                      MOD 2790
P(11,8)=-C24*X(10)                                                      MOD 2800
P(11,9)=C20*X(6)+C19                                                    MOD 2810
P(11,10)=-C20*X(5)-C22*X(7)-C24*X(8)+C65                                MOD 2820
P(11,11)=C22*X(6)+C21                                                   MOD 2830
P(11,12)=C24*X(6)+C23                                                   MOD 2840
P(12,5)=-C29*X(10)                                                      MOD 2850
P(12,6)=C29*X(9)+C31*X(11)+C33*X(12)+C68                                MOD 2860
P(12,7)=-C31*X(10)                                                      MOD 2870
P(12,8)=-C33*X(10)                                                      MOD 2880
P(12,9)=C29*X(6)+C28                                                    MOD 2890
P(12,10)=-C29*X(5)-C31*X(7)-C33*X(8)+C67                                MOD 2900
P(12,11)=C31*X(6)+C30                                                   MOD 2910
P(12,12)=C33*X(6)+C32                                                   MOD 2920
P(13,5)=-C2*X(14)                                                       MOD 2930
P(13,6)=C2*X(13)+C4*X(15)+C6*X(16)+C70                                  MOD 2940
P(13,7)=-C4*X(14)                                                       MOD 2950
P(13,8)=-C6*X(14)                                                       MOD 2960
P(13,9)=-C1*X(14)                                                       MOD 2970
P(13,10)=C1*X(13)+C3*X(15)+C5*X(16)+C69                                 MOD 2980
P(13,11)=-C3*X(14)                                                      MOD 2990
P(13,12)=-C5*X(14)                                                      MOD 3000
P(13,13)=C1*X(10)+C2*X(6)+C1                                            MOD 3010
P(13,14)=-C1*X(9)-C2*X(5)-C3*X(11)-C4*X(7)-C5*X(12)-C6*X(8)+C7          MOD 3020
P(13,15)=C3*X(10)+C4*X(6)+C3                                            MOD 3030
P(13,16)=C5*X(10)+C6*X(6)+C5                                            MOD 3040
P(14,5)=-C11*X(14)                                                      MOD 3050
P(14,6)=C11*X(13)+C13*X(15)+C15*X(16)+C72                               MOD 3060
P(14,7)=-C13*X(14)                                                      MOD 3070
P(14,9)=-C10*X(14)                                                      MOD 3080
P(14,8)=-C15*X(14)                                                      MOD 3090
P(14,10)=C10*X(13)+C12*X(15)+C14*X(16)+C71                              MOD 3100
P(14,11)=-C12*X(14)                                                     MOD 3110
P(14,12)=-C14*X(14)                                                     MOD 3120
P(14,13)=C10*X(10)+C11*X(6)+C10                                         MOD 3130
P(14,14)=-C10*X(9)-C11*X(5)-C12*X(11)-C13*X(7)-C14*X(12)-C15*X(8)+      MOD 3140
1C16                                                                    MOD 3150
```

```
      P(14,15)=C12*X(10)+C13*X(6)+C12
      P(14,16)=C14*X(10)+C15*X(6)+C14
      P(15,5)=-C20*X(14)
      P(15,6)=C20*X(13)+C22*X(15)+C24*X(16)+C74
      P(15,7)=-C22*X(14)
      P(15,8)=-C24*X(14)
      P(15,9)=-C19*X(14)
      P(15,10)=C19*X(13)+C21*X(15)+C23*X(16)+C73
      P(15,11)=-C21*X(14)
      P(15,12)=-C23*X(14)
      P(15,13)=C19*X(10)+C20*X(6)+C19
      P(15,14)=-C19*X(9)-C20*X(5)-C21*X(11)-C22*X(7)-C23*X(12)-C24*X(8)+
     1C25
      P(15,15)=C21*X(10)+C22*X(6)+C21
      P(15,16)=C23*X(10)+C24*X(6)+C23
      P(16,5)=-C29*X(14)
      P(16,6)=C29*X(13)+C31*X(15)+C33*X(16)+C76
      P(16,7)=-C31*X(14)
      P(16,8)=-C33*X(14)
      P(15,9)=-C28*X(14)
      P(15,10)=C28*X(13)+C30*X(15)+C32*X(16)+C75
      P(16,11)=-C30*X(14)
      P(16,12)=-C32*X(14)
      P(15,13)=C28*X(10)+C29*X(6)+C28
      P(16,14)=-C28*X(9)-C29*X(5)-C30*X(11)-C31*X(7)-C32*X(12)-C33*X(8)+
     1C34
      P(16,15)=C30*X(10)+C31*X(6)+C30
      P(16,16)=C32*X(10)+C33*X(6)+C32
    2 ITCON=1
      CALL INVR(P,PINV,16,0,16)
      DO 10 I=1,16
      XINC(I)=0.
      DO 10 J=1,16
   10 XINC(I)=XINC(I)-PINV(I,J)*F(J)
      DO 3 I=1,16
      IF(ABS(XINC(I)).GT.EPS1) ITCON=0
    3 X(I)=X(I)+XINC(I)
   11 CONTINUE
      IF(ITCON.NE.0) GO TO 1
```

```
    4 CONTINUE
      PRINT 500,X,XINC
      GO TO 7
    1 PRINT 400,ITER,X
    7 CONTINUE
      DO 6 I=1,4
      DO 6 J=1,4
    6 A(I,J)=X(4*I-4+J)
      DP=(V*A(3,1)+L*A(2,1))
      DR=(V*A(3,2)+L*A(2,2)+V)
      DBETA=(V*A(3,3)+L*A(2,3)+K)
      DPHI=(V*A(3,4)+L*A(2,4))
      DO 8 I=1,4
      W(I)=(A(I,1)-A(I,2)*DP/DR)*REAL(C(5,1))/PSS+
     1(A(I,3)-A(I,2)*DBETA/DR)*REAL(C(5,2))/BETASS+
     2(A(I,4)-A(I,2)*DPHI/DR)*REAL(C(5,3))/PSS+
     3A(I,2)*REAL(C(5,4))/DR/DSS
    8 W(I)=-W(I)
      Z1=1.-L*A(1,2)/DR
      Z2=-V*A(1,2)/DR
      Z3=-L*A(3,2)/DR
      Z4=1.-V*A(3,2)/DR
      B(1)=(Z4*W(1)-Z2*W(3))/(Z1*Z4-Z2*Z3)
      B(3)=(-Z3*W(1)+Z1*W(3))/(Z1*Z4-Z2*Z3)
      DDELTA=(V*B(3)+L*B(1))
      B(2)=W(2)+DDELTA*A(2,2)/DR
      B(4)=W(4)+DDELTA*A(4,2)/DR
      DP=DP*DSS
      DR=DR*DSS
      DBETA=DBETA*DSS
      DPHI=DPHI*DSS
      PRINT 900,PSS,BETASS,PSS,DP,DR,DBETA,DPHI
      DDELTA=DDELTA*DSS
      PRINT 800,DDELTA
      PRINT 600,((A(I,J),J=1,4),B(I),I=1,4)
      RETURN
  100 FORMAT(8F10.0)
  110 FORMAT(I4,F20.0)
```

```
 200 FORMAT(///,10X,44HAIRPLANE MODEL WITH SPECIFIED TIME HISTORIES,///
    1,10X,29HFLIGHT AND VEHICLE PARAMETERS,//,10X,8HAIRSPEED,F10.1,7H F
    2T/SEC,//,10X,41HCG TO PILOT STATION LONGITUDINAL DISTANCE,F10.2,3H
    3 FT,//,10X,39HDIMENSIONAL CONSTANT FOR DSTAR EQUATION,F10.4,30H CU
    4BIC-FEET/LB-SECONDS-SQUARED,//,10X,16HDYNAMIC PRESSURE,F10.1,14H L
    5B/FT-SQUARED,//,10X,29HROLLRATE NORMALIZATION FACTOR,F10.3,//,10X,
    629HSIDESLIP NORMALIZATION FACTOR,F10.3,//,10X,26HDSTAR NORMALIZATI
    7ON FACTOR,F10.3,/)
 300 FORMAT(10X,43HMAXIMUM NUMBER OF NEWTON-RAPHSON ITERATIONS,I8,/,10X
    1                                        ,17HCONVERGENCE LIMIT,F20.5
    2,//,,4  X ,16F8.4,//)
 400 FORMAT(///,10X,25HITERATIONS TO CONVERGENCE,I8,//,10X,12HN-R SOLUT
    1ION,//,4X,16F8.4,///)
 500 FORMAT(10X,29HN-R SOLUTION DID NOT CONVERGE,/,10X,19HLAST TRIAL SO
    1LUTION,/,4X,16F8.4,/,10X,24HLAST SOLUTION ADJUSTMENT,/,4X,16F8.4)
 600 FORMAT(10X,15HTHE A MATRIX IS,59X,15HTHE B VECTOR IS//,4(10X,4E16.
    16,10X,E16.6//))
 700 FORMAT(//,10X,20HMODIFIED EIGENMATRIX,//,4(10X,4E20.6,//))
 800 FORMAT(   10X,14H(DSTAR)DELTAA=,F20.6,///)
 900 FORMAT(//,10X,19HDISTRIBUTION MATRIX,//,10X,F20.4,//,50X,F20.4,
    1//,70X,F20.4,//,10X,4E20.6,//)
1000 FORMAT(10X,4E20.6)
     END
```

APPENDIX C

Program AANDB Sample Output

DATE= 12/01/75

LIST PARAMETER TABLE 1 1 1 1 1 1 1 1 1 1 1 1 1

CASE NUMBER 1 PLOT CODE 0 TIME RESPONSE CODE 1

INPUT DATA

PN,	0.000	.820	1.020	1.020	1.005	1.030
TIME,	0.00	.50	1.00	1.50	2.00	2.50
BETAN,	0.000	.160	.580	.930	.980	.800
TIME,	0.00	.50	1.00	1.50	2.00	2.50
DSTAR,	0.000	.376	.676	.951	1.260	1.610
TIME,	0.00	.50	1.00	1.50	2.00	2.50

1 1 1 1 1 1 1

1.070	1.090	1.075	1.045	1.025
3.00	3.50	4.00	4.50	5.00
.650	.690	.890	1.090	1.150
3.00	3.50	4.00	4.50	5.00
1.980	2.340	2.650	2.950	3.270
3.00	3.50	4.00	4.50	5.00

	REAL	IMAG	REAL	IMAG
MU,	.159439E+01	0.	.279848E+00	0.
EI,	.9330	0.0000	-2.5470	0.0000

REAL	IMAG	REAL	IMAG
.317662E+00	.749635E+00	.317662E+00	-.749635E+00
-.4112	2.3400	-.4112	-2.3400

	REAL	IMAG	REAL	IMAG
MU,	.932945E+00	0.	-.515220E+01	0.
EI,	-.1388	0.0000	3.2788	6.2832

REAL	IMAG	REAL	IMAG
.497955E+00	.781256E+00	.497955E+00	-.781256E+00
-.1528	2.0067	-.1528	-2.0067

	REAL	IMAG	REAL	IMAG
MU.	.103636E+01	0.	.392694E+00	0.
EI.	.0714	0.0000	-1.8694	0.0000

REAL	IMAG	REAL	IMAG
.256118E+00	.105674E+01	.256118E+00	-.105674E+01
.1675	2.6660	.1675	-2.6660

MU VALUES SPECIFIED

	REAL	IMAG	REAL	IMAG
MU,	.300517E+00	0.	.998453E+00	0.
EIGEN,	-2.4045	0.0000	-.0031	0.0000

REAL	IMAG	REAL	IMAG
.451562E+00	-.756023E+00	.451562E+00	.756023E+00
-.2543	-2.0648	-.2543	2.0648

THE GENERATED PHI DATA IS,

PHIN,	0.000	.247	.721	1.235	1.740	2.247
TIME,	0.00	.50	1.00	1.50	2.00	2.50
		2.772	3.313	3.856	4.386	4.905
		3.00	3.50	4.00	4.50	5.00

TIME RESPONSE COEFFICIENTS

	REAL	IMAG	REAL	IMAG	REAL	IMAG
PN,	-1.1111	-.0000	2.1496	.0000	.0155	.0414
BETAN,	-.1199	-.0000	-32.9159	.0000	-.1988	-.1282
PHIN,	.4435	-.0000	-345.4494	.0000	-.0190	.0034
DSTAR,	-.0086	-.0000	-215.9887	.0000	.0179	.0368

REAL	IMAG	REAL	IMAG
.0155	-.0414	-1.0695	-.0000
-.1988	.1282	33.4334	-.0000
-.0190	-.0034	345.0439	-.0000
.0179	-.0368	215.9615	-.0000

FITTING RESULTS

	REAL	IMAG	REAL	IMAG	REAL	IMAG
MU,	.3005	0.0000	.9985	0.0000	.4516	-.7560
EIGEN,	-2.4045	0.0000	-.0031	0.0000	-.2543	-2.0648
COEFFICIENTS						
PN,	-1.1111	-.0000	2.1496	.0000	.0155	.0414
BETAN,	-.1199	-.0000	-32.9159	.0000	-.1988	-.1282
PHIN,	.4435	-.0000	-345.4494	.0000	-.0190	.0034
DSTAR,	-.0086	-.0000	-215.9887	.0000	.0179	.0368

REAL	IMAG	REAL	IMAG
.4516	.7560		
-.2543	2.0648		
.0155	-.0414	-1.0695	-.0000
-.1988	.1282	33.4334	-.0000
-.0190	-.0034	345.0439	-.0000
.0179	-.0368	215.9615	-.0000

AIRPLANE MODEL WITH SPECIFIED TIME HISTORIES

FLIGHT AND VEHICLE PARAMETERS

AIRSPEED 612.2 FT/SEC

CG TO PILOT STATION LONGITUDINAL DISTANCE 22.24 FT

DIMENSIONAL CONSTANT FOR DSTAR EQUATION -.3190 CUBIC-FEET/LB-SECONDS-SQUARED

DYNAMIC PRESSURE 331.8 LB/FT-SQUARED

ROLLRATE NORMALIZATION FACTOR .500

SIDESLIP NORMALIZATION FACTOR 10.000

DSTAR NORMALIZATION FACTOR .010

MAXIMUM NUMBER OF NEWTON-RAPHSON ITERATIONS 50
CONVERGENCE LIMIT .000010

X 1.0000 0.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000
1.0000 0.0000 0.0000 0.0000 0.0000 1.0000

ITERATIONS TO CONVERGENCE 3

N-R SOLUTION

```
-2.3588   .7708-10.9390  -.0026  -.0534  -.3587  3.8479  -.0002   .0254 -1.0051
                                 -.2013   .0533   .9596  -.0208   .2782   .0026
```

DISTRIBUTION MATRIX

.5000			
		10.0000	
			.5000
.143537E+00	-.111278E+00	-.143513E+01	.326127E+00

(DSTAR)DELTAA= -.023401

ORIGINAL PAGE IS
OF POOR QUALITY

THE A MATRIX IS

-.235877E+01	.770832E+00	-.109390E+02	-.255620E-02
-.534355E-01	-.358690E+00	.384788E+01	-.206077E-03
.253874E-01	-.100515E+01	-.201315E+00	.532788E-01
.959579E+00	-.207575E-01	.278157E+00	.262703E-02

THE B VECTOR IS

.581800E+01
-.527309E-01
-.215179E+00
.495307E-01

RESPONSE ENVELOPES

TIME	PN		BETAN		DSTAR	
0.00	0.000	0.000	2.000	-2.000	4.000	-4.000
.10	.333	.010	2.000	-2.000	4.000	-4.000
.20	.667	.027	2.000	-2.000	4.000	-4.000
.30	1.000	.065	2.000	-2.000	4.000	-4.000
.40	1.043	.107	2.000	-2.000	4.000	-4.000
.50	1.064	.160	2.000	-2.000	4.000	-4.000
.60	1.076	.227	2.000	-2.000	4.000	-4.000
.70	1.080	.300	2.000	-2.000	4.000	-4.000
.80	1.080	.360	2.000	-2.000	4.000	-4.000
.90	1.078	.409	2.000	-2.000	4.000	-4.000
1.00	1.076	.450	2.000	-2.000	4.000	-4.000
1.10	1.074	.479	2.000	-2.000	4.000	-4.000
1.20	1.072	.507	2.000	-2.000	4.000	-4.000
1.30	1.070	.530	2.000	-2.000	4.000	-4.000
1.40	1.069	.556	2.000	-2.000	4.000	-4.000
1.50	1.067	.579	2.000	-2.000	4.000	-4.000
1.60	1.065	.600	2.000	-2.000	4.000	-4.000
1.70	1.063	.620	2.000	-2.000	4.000	-4.000
1.80	1.061	.639	2.000	-2.000	4.000	-4.000
1.90	1.059	.656	2.000	-2.000	4.000	-4.000
2.00	1.057	.677	2.000	-2.000	4.000	-4.000
2.10	1.055	.692	2.100	-2.100	4.200	-4.200
2.20	1.053	.708	2.200	-2.200	4.400	-4.400
2.30	1.051	.722	2.300	-2.300	4.600	-4.600
2.40	1.050	.733	2.400	-2.400	4.800	-4.800
2.50	1.048	.748	2.500	-2.500	5.000	-5.000
2.60	1.046	.762	2.600	-2.600	5.200	-5.200
2.70	1.044	.778	2.700	-2.700	5.400	-5.400
2.80	1.042	.790	2.800	-2.800	5.600	-5.600
2.90	1.040	.804	2.900	-2.900	5.800	-5.800
3.00	1.038	.816	3.000	-3.000	6.000	-6.000
3.10	1.036	.829	3.100	-3.100	6.200	-6.200
3.20	1.034	.840	3.200	-3.200	6.400	-6.400
3.30	1.032	.852	3.300	-3.300	6.600	-6.600
3.40	1.030	.861	3.400	-3.400	6.800	-6.800
3.50	1.029	.871	3.500	-3.500	7.000	-7.000
3.60	1.027	.880	3.600	-3.600	7.200	-7.200
3.70	1.025	.888	3.700	-3.700	7.400	-7.400
3.80	1.023	.896	3.800	-3.800	7.600	-7.600
3.90	1.021	.908	3.900	-3.900	7.800	-7.800
4.00	1.019	.915	4.000	-4.000	8.000	-8.000
4.10	1.017	.920	4.100	-4.100	8.200	-8.200
4.20	1.015	.929	4.200	-4.200	8.400	-8.400
4.30	1.013	.938	4.300	-4.300	8.600	-8.600
4.40	1.011	.947	4.400	-4.400	8.800	-8.800
4.50	1.010	.956	4.500	-4.500	9.000	-9.000
4.60	1.008	.965	4.600	-4.600	9.200	-9.200

ORIGINAL PAGE IS
OF POOR QUALITY

4.70	1.006	.974	4.700	-4.700	9.400	-9.400
4.80	1.004	.983	4.800	-4.800	9.600	-9.600
4.90	1.002	.992	4.900	-4.900	9.800	-9.800
5.00	1.000	1.000	5.000	-5.000	10.000	-10.000

PNDOT		BETANDOT		OSTARDOT	
4.000	.806	6.000	-6.000	12.000	-12.000
4.000	.711	6.000	-6.000	12.000	-12.000
1.686	.255	4.800	-4.800	9.660	-9.600
1.122	-.224	4.150	-4.150	8.300	-8.300
.992	-.297	3.720	-3.720	7.440	-7.440
.908	-.323	3.280	-3.280	6.560	-6.560
.843	-.327	2.910	-2.910	5.820	-5.820
.797	-.323	2.610	-2.610	5.220	-5.220
.724	-.319	2.370	-2.370	4.740	-4.740
.663	-.304	2.100	-2.100	4.200	-4.200
.613	-.289	1.950	-1.950	3.900	-3.900
.556	-.274	1.780	-1.780	3.560	-3.560
.510	-.251	1.610	-1.610	3.220	-3.220
.464	-.228	1.490	-1.490	2.980	-2.980
.421	-.205	1.350	-1.350	2.700	-2.700
.379	-.183	1.260	-1.260	2.520	-2.520
.349	-.148	1.170	-1.170	2.340	-2.340
.318	-.129	1.120	-1.120	2.240	-2.240
.295	-.125	1.070	-1.070	2.140	-2.140
.272	-.118	1.050	-1.050	2.100	-2.100
.257	-.125	1.000	-1.000	2.000	-2.000
.234	-.123	1.000	-1.000	2.000	-2.000
.222	-.121	1.000	-1.000	2.000	-2.000
.211	-.120	1.000	-1.000	2.000	-2.000
.199	-.118	1.000	-1.000	2.000	-2.000
.192	-.116	1.000	-1.000	2.000	-2.000
.188	-.114	1.000	-1.000	2.000	-2.000
.184	-.112	1.000	-1.000	2.000	-2.000
.180	-.110	1.000	-1.000	2.000	-2.000
.176	-.109	1.000	-1.000	2.000	-2.000
.172	-.107	1.000	-1.000	2.000	-2.000
.168	-.105	1.000	-1.000	2.000	-2.000
.165	-.103	1.000	-1.000	2.000	-2.000
.169	-.102	1.000	-1.000	2.000	-2.000
.165	-.100	1.000	-1.000	2.000	-2.000
.165	-.098	1.000	-1.000	2.000	-2.000
.157	-.096	1.000	-1.000	2.000	-2.000
.142	-.095	1.000	-1.000	2.000	-2.000
.119	-.093	1.000	-1.000	2.000	-2.000
.107	-.091	1.000	-1.000	2.000	-2.000
.096	-.089	1.000	-1.000	2.000	-2.000
.080	-.088	1.000	-1.000	2.000	-2.000
.073	-.086	1.000	-1.000	2.000	-2.000
.069	-.084	1.000	-1.000	2.000	-2.000
.061	-.082	1.000	-1.000	2.000	-2.000
.054	-.081	1.000	-1.000	2.000	-2.000
.046	-.079	1.000	-1.000	2.000	-2.000
.046	-.077	1.000	-1.000	2.000	-2.000
.046	-.075	1.000	-1.000	2.000	-2.000

.046	-.074	1.000	-1.000	2.000	-2.000
.046	-.072	1.000	-1.000	2.000	-2.000

FITTED TIME RESPONSES

TIME	PN	BETAN	DSTAR	PNDOT	BETANDOT	DSTARDOT
0.00	.000	-.000	-.000	2.828	-.038	.832
.10	.252	.003	.082	2.232	.097	.803
.20	.450	.020	.160	1.754	.245	.770
.30	.606	.052	.236	1.369	.395	.736
.40	.727	.099	.308	1.058	.538	.702
.50	.819	.159	.376	.806	.665	.668
.60	.889	.231	.441	.601	.769	.637
.70	.941	.312	.504	.436	.846	.609
.80	.978	.399	.563	.304	.893	.585
.90	1.003	.489	.621	.199	.907	.566
1.00	1.018	.579	.677	.117	.890	.553
1.10	1.027	.666	.732	.055	.843	.545
1.20	1.030	.747	.786	.009	.769	.542
1.30	1.029	.819	.840	-.021	.671	.542
1.40	1.026	.881	.895	-.040	.556	.545
1.50	1.021	.930	.951	-.048	.428	.553
1.60	1.017	.966	1.009	-.048	.294	.566
1.70	1.012	.989	1.068	-.041	.158	.582
1.80	1.009	.998	1.129	-.029	.028	.601
1.90	1.006	.995	1.192	-.013	-.093	.621
2.00	1.006	.980	1.257	.004	-.199	.643
2.10	1.007	.955	1.325	.022	-.288	.664
2.20	1.010	.923	1.394	.039	-.355	.684
2.30	1.015	.885	1.465	.055	-.400	.703
2.40	1.021	.844	1.538	.068	-.421	.719
2.50	1.028	.802	1.611	.078	-.419	.731
2.60	1.037	.761	1.686	.085	-.396	.741
2.70	1.045	.723	1.761	.088	-.352	.746
2.80	1.054	.691	1.835	.087	-.292	.748
2.90	1.063	.665	1.910	.082	-.218	.746
3.00	1.070	.648	1.983	.075	-.134	.741
3.10	1.077	.639	2.056	.064	-.045	.733
3.20	1.083	.639	2.128	.051	.046	.722
3.30	1.087	.648	2.198	.037	.135	.709
3.40	1.090	.666	2.267	.021	.218	.695
3.50	1.092	.691	2.334	.005	.292	.680
3.60	1.091	.724	2.400	-.011	.355	.665
3.70	1.090	.762	2.464	-.025	.403	.651
3.80	1.086	.804	2.527	-.038	.437	.638
3.90	1.082	.848	2.590	-.049	.454	.626
4.00	1.077	.894	2.651	-.058	.456	.617
4.10	1.071	.939	2.712	-.064	.443	.609
4.20	1.064	.982	2.772	-.067	.416	.605
4.30	1.057	1.022	2.832	-.068	.377	.603
4.40	1.050	1.057	2.893	-.066	.329	.603
4.50	1.044	1.088	2.953	-.062	.273	.606
4.60	1.038	1.112	3.015	-.055	.213	.611

4.70	1.033	1.130	3.077	-.047	.151	.625
4.80	1.029	1.142	3.140	-.037	.090	.634
4.90	1.026	1.148	3.204	-.027	.032	.644
5.00	1.024	1.149	3.269	-.016	-.019	.654

PAYNTERS RECIPE NUMBER IS, 19

DIFFERENCE EQUATION P MATRIX,

.788	.117	-.944	-.003
-.004	.946	.374	.001
.003	-.097	.960	.005
.085	.002	-.021	1.000

DIFFERENCE EQUATION Q VECTOR,

.528
-.011
-.020
.031

INTEGRATED RESPONSE

TIME	PN	BETAN	DSTAR
0.00	0.000	0.000	-.000
.10	.252	.003	.082
.20	.450	.020	.160
.30	.606	.052	.236
.40	.727	.099	.308
.50	.819	.159	.376
.60	.889	.231	.441
.70	.941	.312	.504
.80	.978	.399	.563
.90	1.003	.489	.521
1.00	1.018	.579	.677
1.10	1.027	.666	.732
1.20	1.030	.747	.786
1.30	1.029	.819	.840
1.40	1.026	.881	.895
1.50	1.021	.930	.951
1.60	1.017	.966	1.009
1.70	1.012	.989	1.068
1.80	1.009	.998	1.129
1.90	1.006	.995	1.192
2.00	1.006	.980	1.257
2.10	1.007	.955	1.325
2.20	1.010	.923	1.394
2.30	1.015	.885	1.465
2.40	1.021	.844	1.538
2.50	1.028	.802	1.611
2.60	1.037	.761	1.686
2.70	1.045	.723	1.761
2.80	1.054	.691	1.835
2.90	1.063	.665	1.910
3.00	1.070	.648	1.983
3.10	1.077	.639	2.056
3.20	1.083	.639	2.128
3.30	1.087	.648	2.198
3.40	1.090	.666	2.267
3.50	1.092	.691	2.334
3.60	1.091	.724	2.400
3.70	1.090	.762	2.464
3.80	1.086	.804	2.527
3.90	1.082	.848	2.590
4.00	1.077	.894	2.651
4.10	1.071	.939	2.712
4.20	1.064	.982	2.772
4.30	1.057	1.022	2.832
4.40	1.050	1.057	2.893
4.50	1.044	1.088	2.953
4.60	1.038	1.112	3.015

4.70	1.033	1.130	3.077
4.80	1.029	1.142	3.140
4.90	1.026	1.148	3.204
5.00	1.024	1.149	3.269

FIRST DERIVATIVES OF INTEGRATED RESPONSES

TIME	PNDOT	BETANDOT	DSTARDOT
0.00	2.828	-.038	.832
.10	2.232	.097	.803
.20	1.754	.245	.770
.30	1.369	.395	.736
.40	1.058	.538	.702
.50	.806	.665	.668
.60	.601	.769	.637
.70	.436	.846	.609
.80	.304	.893	.585
.90	.199	.907	.566
1.00	.117	.890	.553
1.10	.055	.843	.545
1.20	.009	.769	.542
1.30	-.021	.671	.545
1.40	-.040	.556	.553
1.50	-.048	.428	.566
1.60	-.048	.294	.582
1.70	-.041	.158	.601
1.80	-.029	.028	.621
1.90	-.013	-.093	.643
2.00	.004	-.199	.664
2.10	.022	-.288	.684
2.20	.039	-.355	.703
2.30	.055	-.400	.719
2.40	.068	-.421	.731
2.50	.078	-.419	.741
2.60	.085	-.396	.746
2.70	.088	-.352	.748
2.80	.087	-.292	.746
2.90	.082	-.218	.741
3.00	.075	-.134	.733
3.10	.064	-.045	.722
3.20	.051	.046	.709
3.30	.037	.135	.695
3.40	.021	.218	.680
3.50	.005	.292	.665
3.60	-.011	.355	.651
3.70	-.025	.403	.638
3.80	-.038	.437	.626
3.90	-.049	.454	.617
4.00	-.058	.456	.609
4.10	-.064	.443	.605
4.20	-.067	.416	.603
4.30	-.068	.377	.603
4.40	-.066	.329	.606
4.50	-.062	.273	.611
4.60	-.055	.213	.617

4.70	-.047	.151	.625
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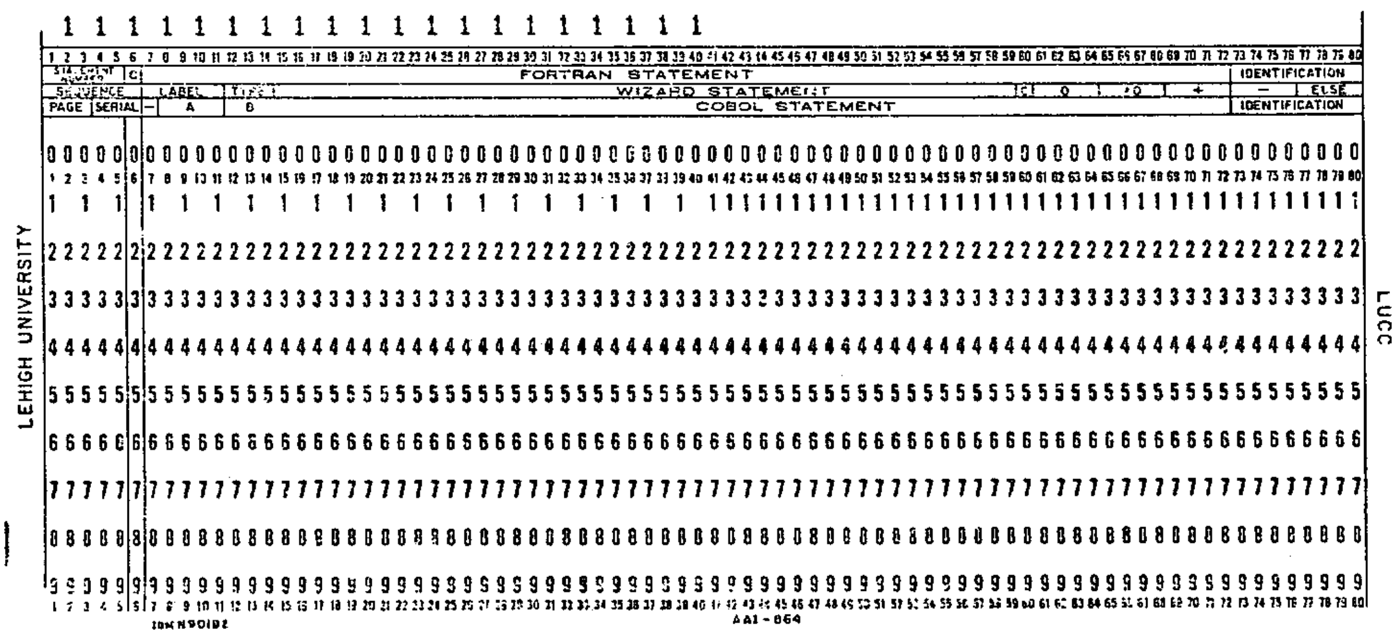
APPENDIX D

Listing of Program AANDB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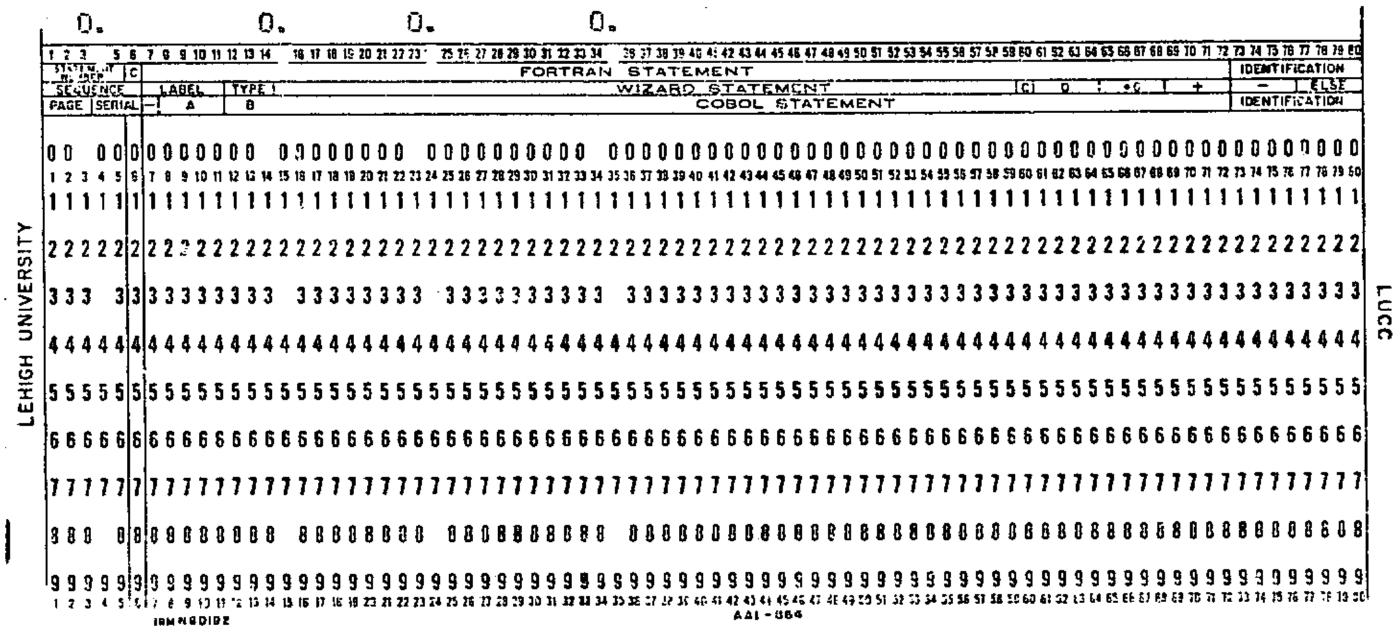
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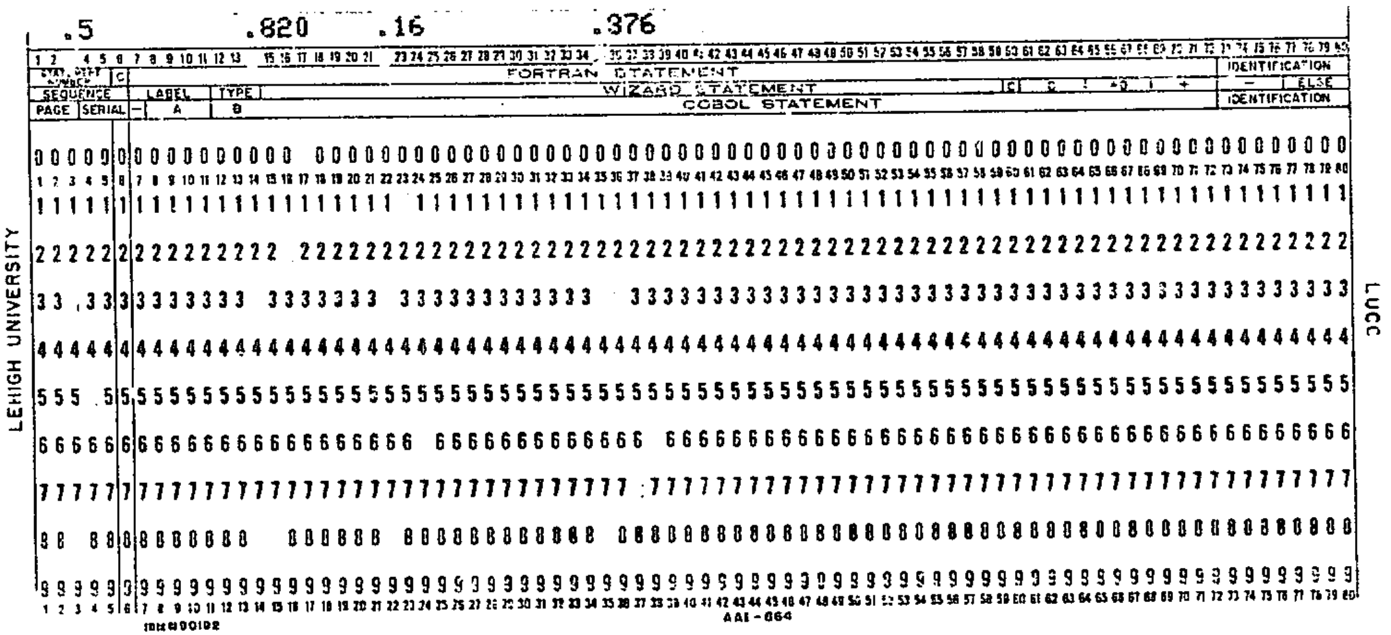
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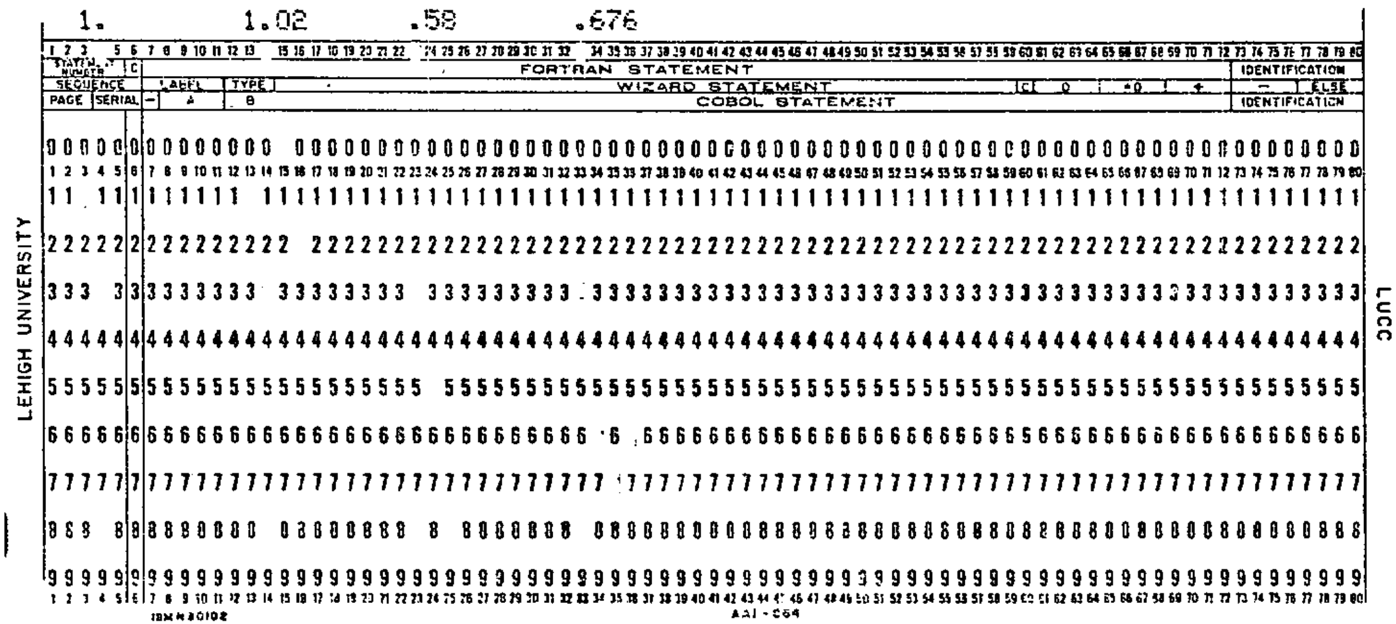
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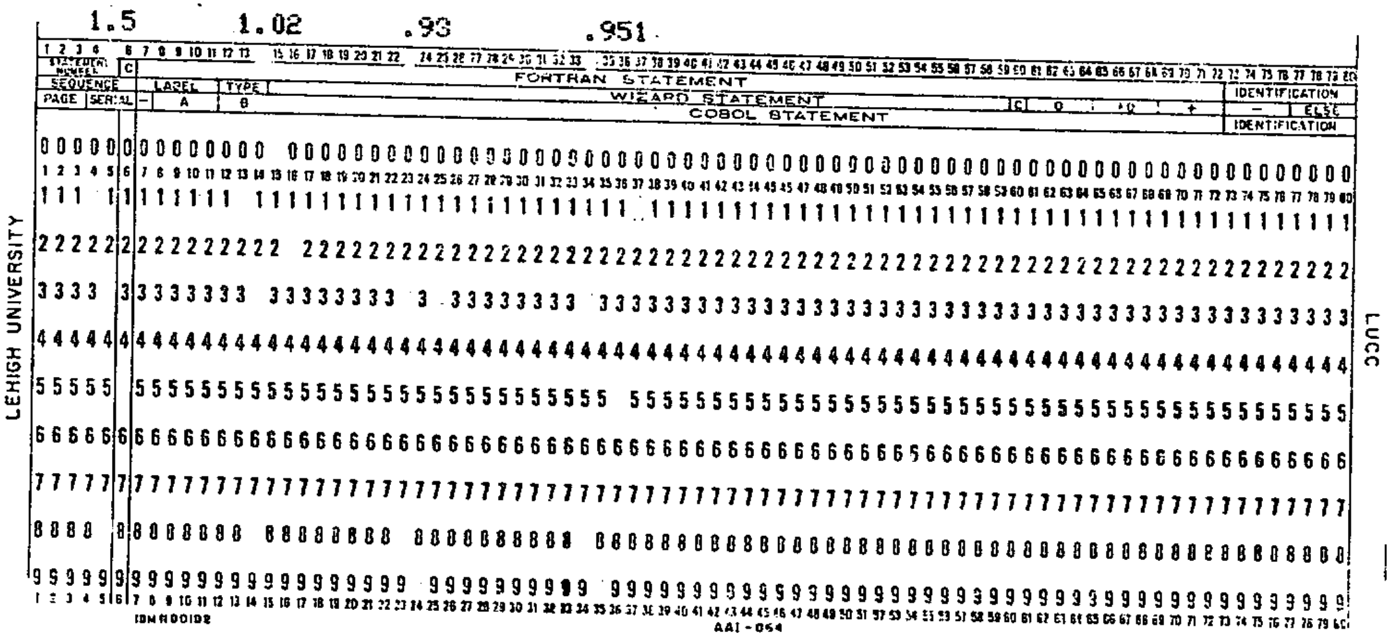
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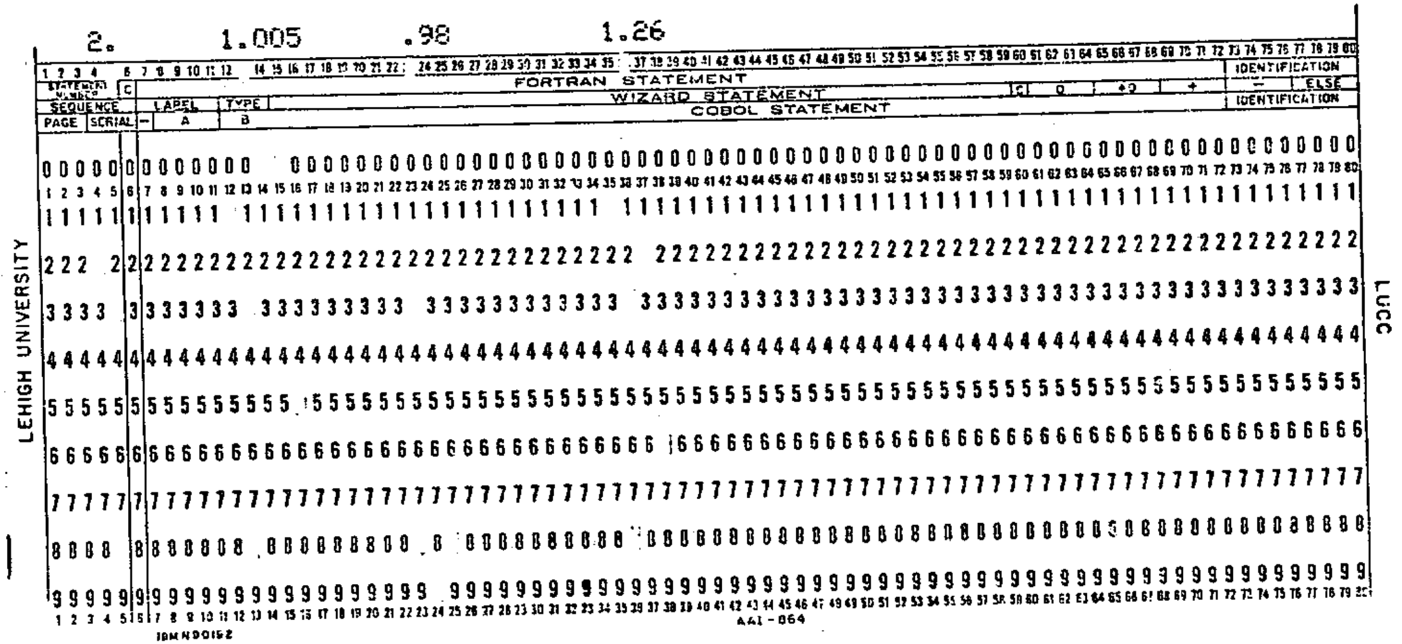
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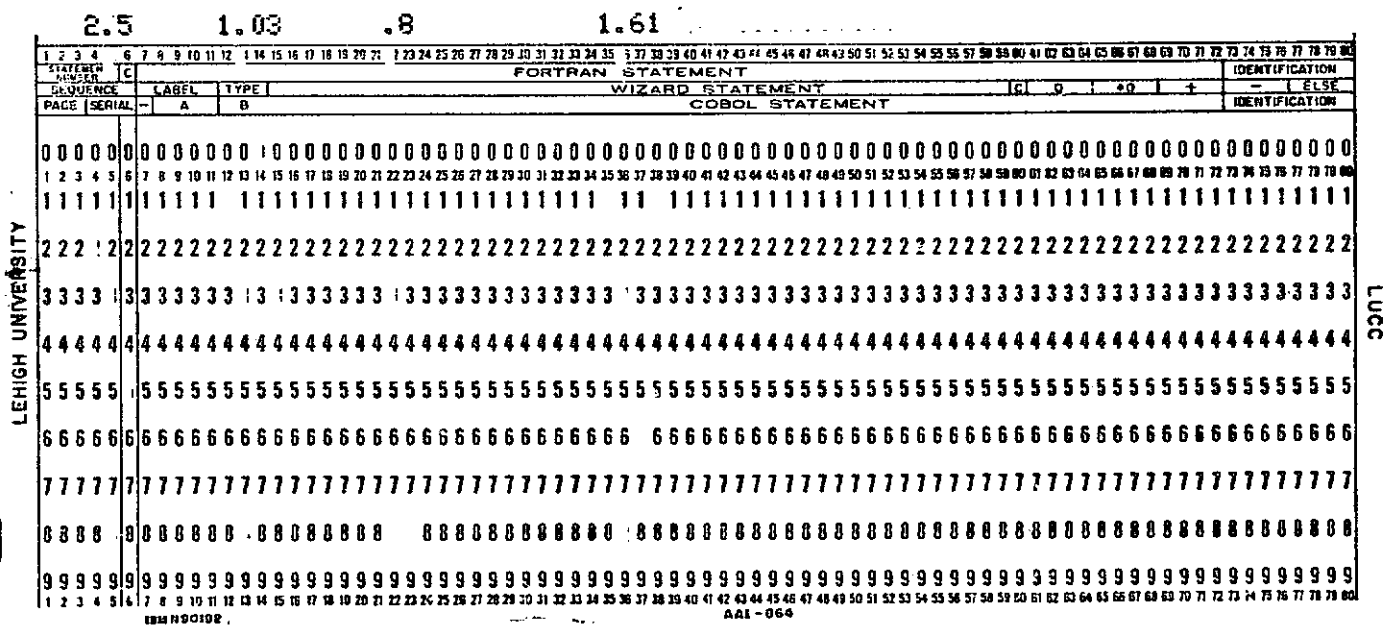
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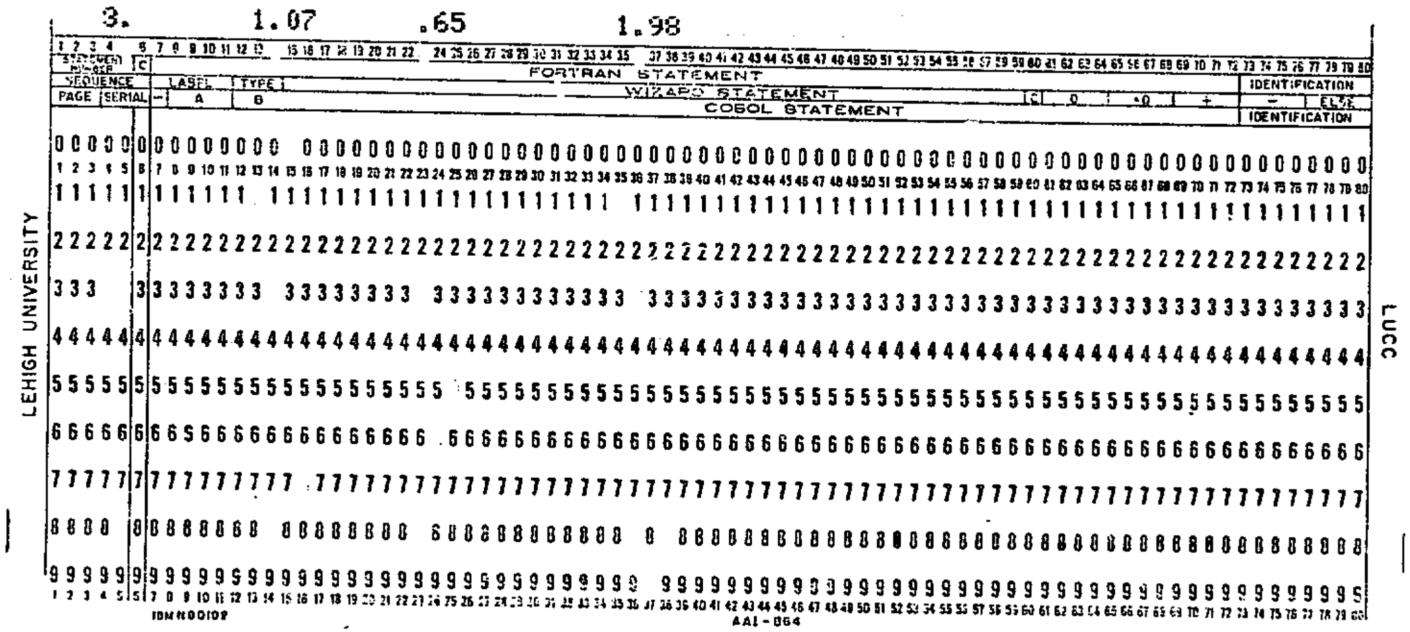
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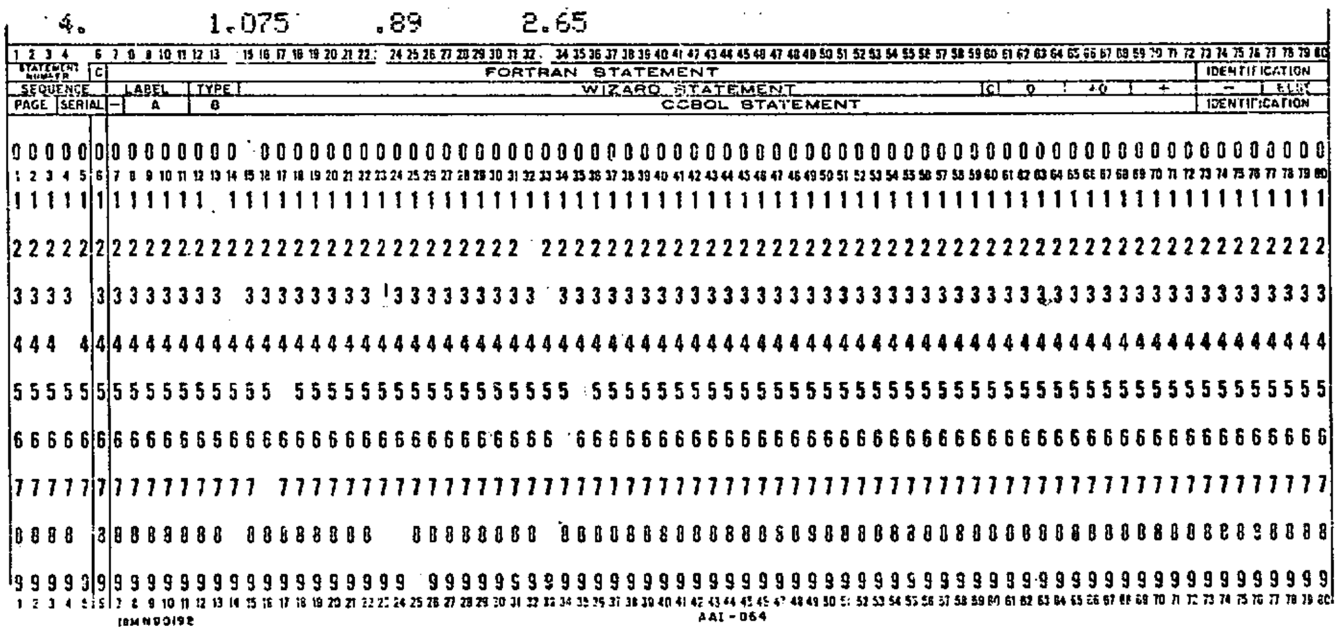
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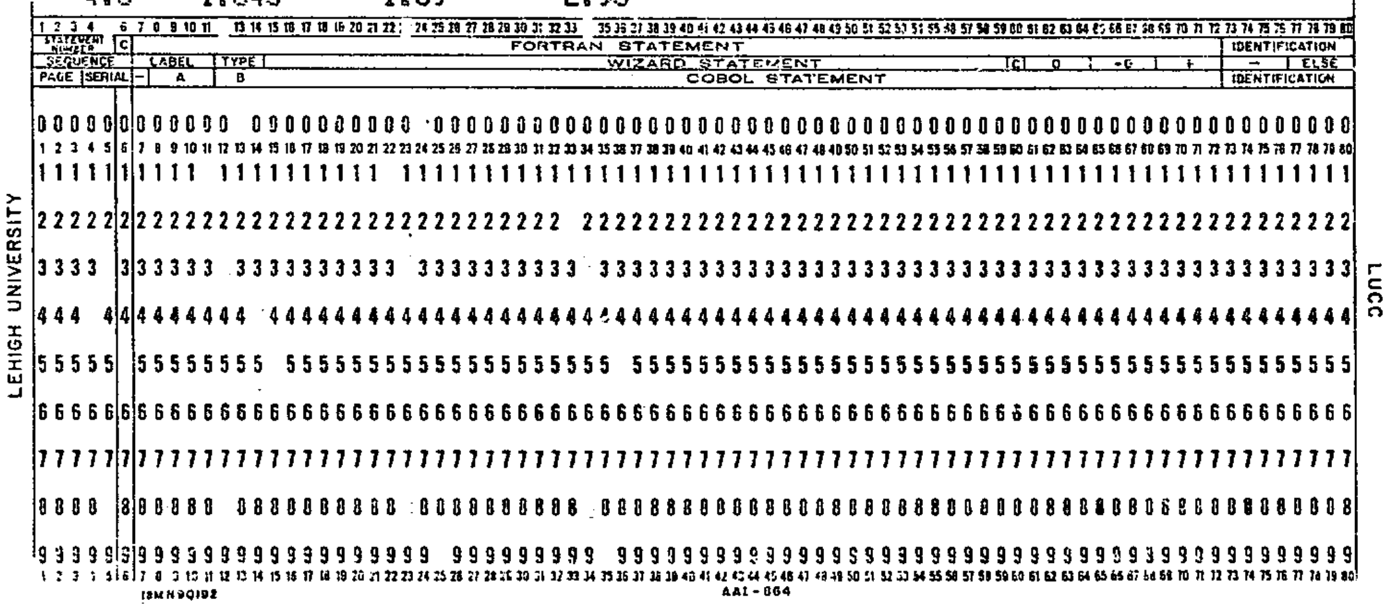

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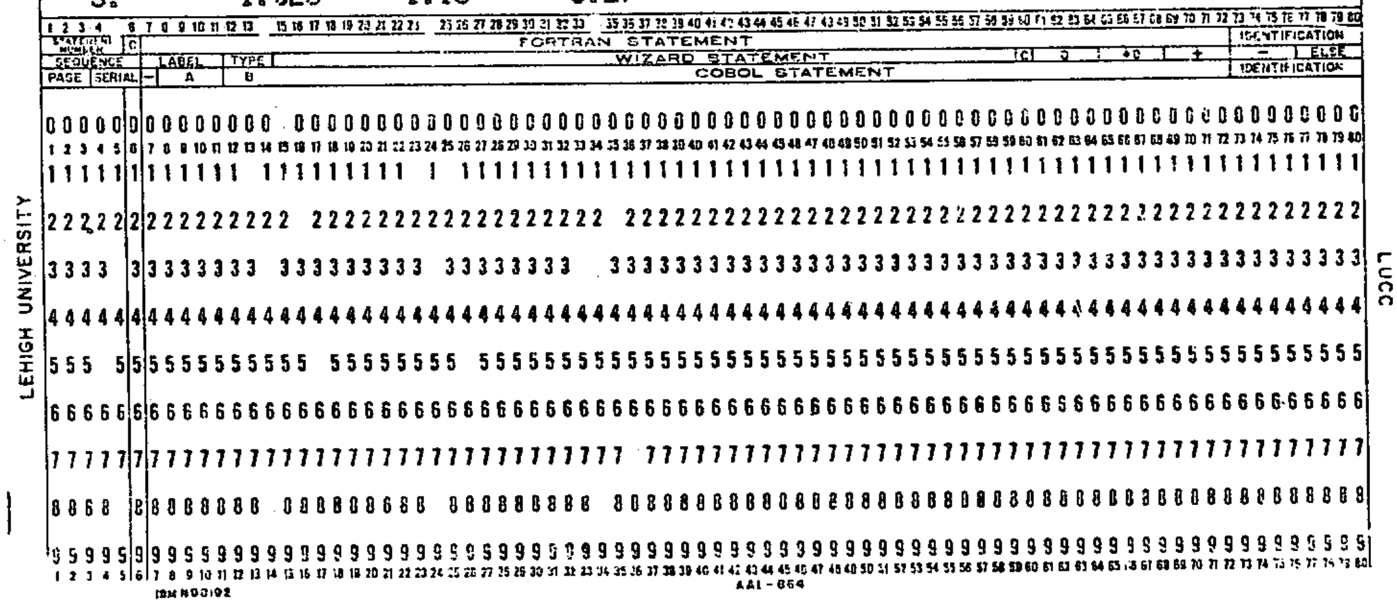

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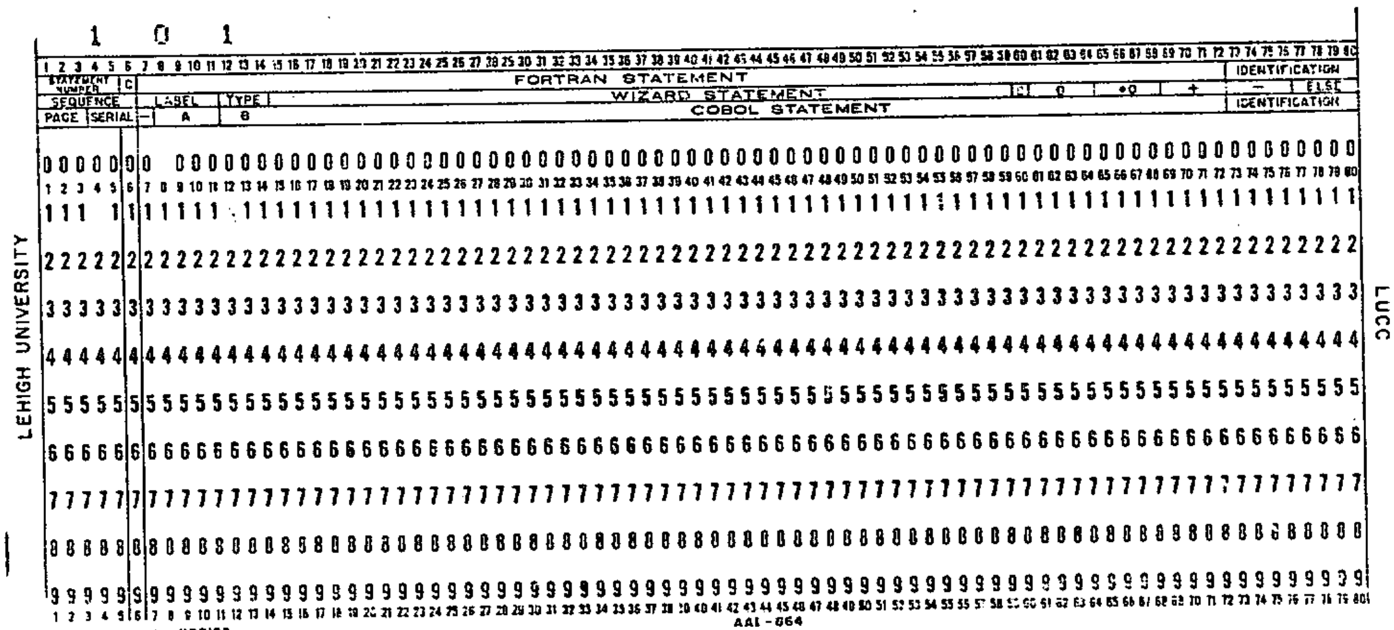

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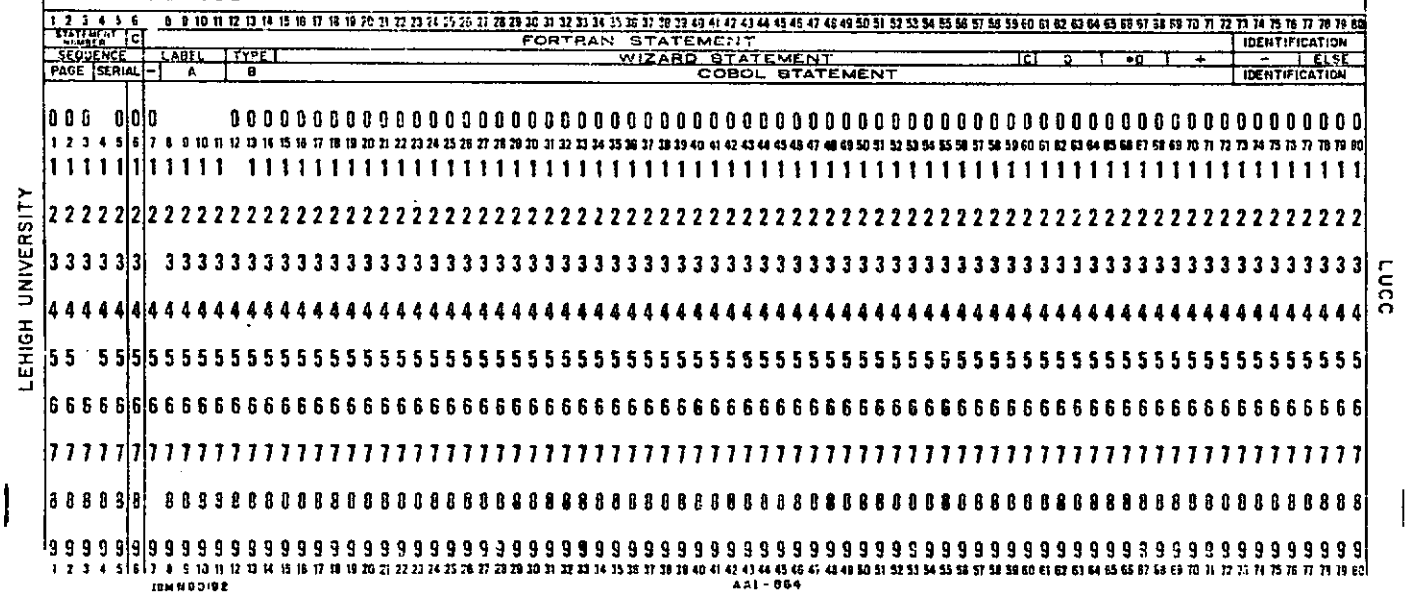
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9. Private communication between the author and Harriet Smith, NASA Flight Research Center, Edwards, CA.